2

Fundamentals of Electrical Measurements

2.1 Main methods of measurement

The Oxford Dictionary explains the term *measure* as "*ascertain the size, amount or degree of (something) by using an instrument or device marked in standard units or by comparing it with an object of known size*" (from the Latin *mensurare* – to measure)[1].

More professional sounds following definition: *The measurement is a cognitive process of gathering the information from the physical world. In this process a value of a quantity is determined (in defined time and conditions) by comparison it (with known uncertainty) with the standard reference value.*[2]

In this definition we can emphasize several important factors. We see that always exists **standard** of measured value. It is not necessary to include such standard to the measuring device because this device can be calibrated (scaled, tested, standardized) by comparison with more accurate device. But always on the top of this pyramid we can find international standard of this physical value. This problem we call as *traceability* – unbroken chain between main standard and individual measuring instrument (this problem is discussed later in more details).

Other important factor is the **uncertainty** of measurements. We never know estimated value without any error (although it can be sufficiently small). Therefore we always are obliged to consider accuracy of measurement. Also this problem is discussed in more details later.

Another important factor is statement that we perform measurement in defined time and conditions. It means that we should take into consideration that many investigated processes are dynamic – changing in time. Moreover measurements are not performed in isolated environment. They can be disturbed by external interferences (for example external electromagnetic field) as well as the variation of external conditions (for example influence of temperature).

In this chapter we discuss still other factor – measurement is a **comparison** with other more accurate (assumed as standard) value. This process of comparison can be realized in different ways:

- by compensation (subtraction) (Figure 2.1a),
- by comparator principle (Figure 2.1b),
- by substitution (Figure 2.1c)

or very often:

- by conversion (Figure 2.1d).

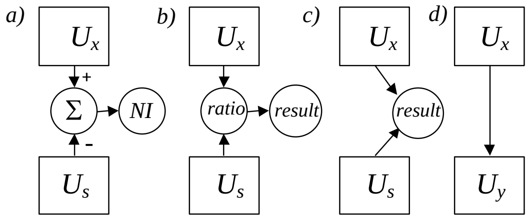

FIGURE 2.1
Various methods of comparison of two values: a) compensation, b) comparatopr, c) substitution, d) conversion.

The first method is obvious. We substrate from measured value X the standard value X_s and observe the difference (using null indicator NI). When both values are exactly the same (compensated each other) the null indictor points zero. We can simply perform such operation by changing reference value and observing null indicator.

The example of the compensation measuring device known as potentiometer (the main measuring instrument in old times) is presented in Fig. 2.2. The measured voltage U_x is compensated by voltage drop I_sR_x on adjustable resistor R. The measurement was performed in two steps. In the first step the standard voltage source U_s (for example $U_s = 1.01805\ V$) was connected instead of measured voltage. If we adjust the resistor to the value defined by standard voltage (in our case $R_s = 1.01805\ k\Omega$) and calibrate the current I_s to obtain zero signal on null indicator we can say that we

[1] The most of terms related to measurements are defined by "International Vocabulary of Basic and General Terms in Metrology – ISO VIM", International Organization for Standardization ISO, Geneva, 1993 (revised edition 2004).

[2] The International ISO Vocabulary proposes following definitions: Measurement is a process of experimentally obtaining information about the magnitude of a quantity. Measurement implies a measurement procedure based on a theoretical model. In practice measurement presupposes a calibrated measuring system, possibly subsequently verified. The measurement can change the phenomenon, body or substance under study such that the quantity that is actually measured differs from the value intended to be measured and called the measurand.

have standard value of current (in our case *1 mA*). In the next step we can precisely determine measured voltage from the value of the resistor R_x (of course in this second step we again look for zero state of null indicator).

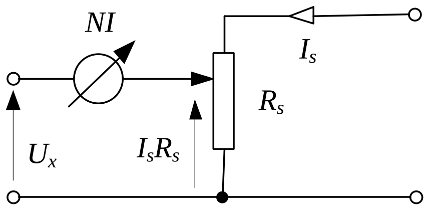

FIGURE 2.2
The principle of operation of the potentiometer device.

The conditions of equilibrium are as follows:

$$I_s R_s - U_s = 0 \tag{2.1}$$

for the first step, and:

$$I_s R_x - U_x = 0 \tag{2.2}$$

Thus

$$U_x = U_s \frac{1}{R_s} R_x \tag{2.3}$$

Why we had to perform such complicated operation instead of simply compensation two voltages? Because a long time ago we did not have adjustable voltage source. In contrary the resistance is very easy to adjust and moreover we are able to prepare it with extreme high accuracy.

Presented potentiometer device exhibits two very important advantages. First of all because accuracy depends only on the accuracy of resistor R_x (see Eq. 2.3) we are able to determine the voltage also with high accuracy (indeed the potentiometer devices were earlier the most accurate "standard" measuring instruments – with accuracy even better than 0.01%)[3].

The second important advantage lies in the idea of compensation. In measurements the best case is if measuring device does not influence estimated result. In the case of measurement of voltage it means that voltmeter should exhibit resistance as high as possible. Because in the state of equilibrium a null indicator informs that we do not consume current from measured source it means that the resistance is close to infinity.

Recently an old potentiometer is in museum. But this idea is still valid due to its important advantages (high accuracy and high input resistance). Figure 2.3 presents modern realization of the compensation principle.

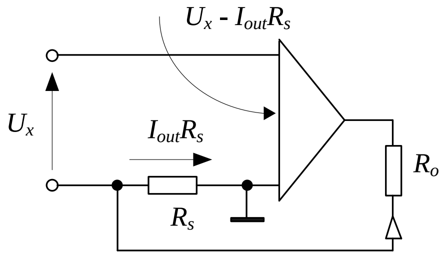

FIGURE 2.3
The autocompensation device.

A long time ago as null indicator commonly was used very sensitive (specially designed) pointing devices, known as *galvanometer*. Recently galvanometer can be easy substituted by very sensitive amplifier.

In the compensation circuit presented in Figure 2.2 the balance process was performed manually. In modern electronic circuits this compensation can be realized by feedback. An example is presented in Figure 2.3. When input voltage is equal to zero also input voltage of the amplifier (as well as output current) are equal to zero. When input voltage increases it causes that in the input of amplifier appears (the same output current increases). This current results in voltage drop on resistance R_s. Due to feedback this voltage drop is subtracted from the input voltage and the equilibrium condition is:

$$U_x - I_{out} R_s = 0 \tag{2.4}$$

And next:

$$I_{out} = \frac{1}{R_s} U_x \tag{2.5}$$

In the equilibrium the circuit is auto-compensated. It means that we profit all advantages of compensation principle: very high input resistance and very high accuracy (transfer coefficient *K = out/in* depends only on the value of resistor R_s that we can prepare with high accuracy).

But *feedback* introduces also many additional advantages. In Eq. 2.5 does not exist such factors as amplification coefficient K_u of amplifier as well as load

[3] By measurement of voltage drop on standard resistor we can determine current with high accuracy by using voltage potentiometer. And next knowing current and voltage we were able to determine resistance.

resistor R_0. It means that changes of amplification (for example by aging or by influence of temperature) do not influence the accuracy. If load resistance also does not influence the accuracy we can assume that our measuring circuit acts as current source – we simply convert voltage into current. The current output signal is valuable taking into account signal transmission. During such transmission the resistance of connecting wires depends on the temperature changes what can influence the output signal. But if we have current output this signal does not depend on these changes of resistance.

Other benefit of feedback is improvement of linearity. Every amplifier has nonlinear transfer characteristic *out/in* because for large signal we are close to saturation. As larger is input signal as larger is error of nonlinearity. But if we apply feedback the input signal of amplifier is close zero – thus we are far from nonlinear part of characteristics.

In the device presented in Figure 2.3 we compensate two analogue values. But compensation principle is also very useful in digital circuits. In digital technique as null indicator commonly is used special amplifier known as *comparator*. The principle of operation of comparator is presented in Figure 2.4.

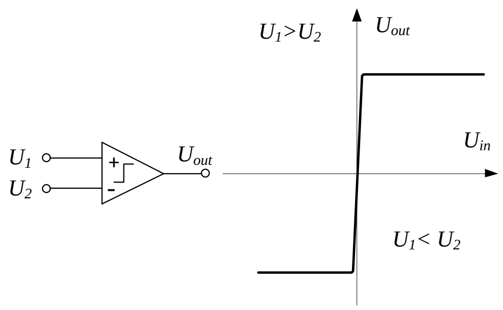

FIGURE 2.4
The principle of operation of the comparator.

The comparator is a differential amplifier with output voltage U_{out} depending on the difference of two input voltages U_1 and U_2:

$$U_{out} = K_u \left(U_1 - U_2\right) \tag{2.6}$$

When amplification coefficient K_u of amplifier is very large even small difference between two input voltages causes saturation of amplifier. Therefor this device is switching the output voltage between ± saturation voltages and this way is indicated zero voltage (if the second input is grounded - thus the second voltage is zero).

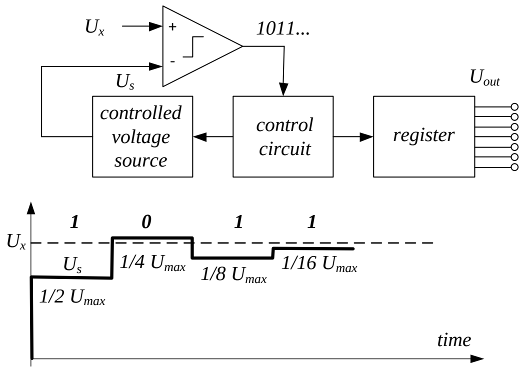

FIGURE 2.5
The digital compensation device - SAR.

Figure 2.5 presents the digital device basing on compensation principle – known as SAR analaog-to-digital converter (*Successive Approximation Register*). The standard voltage source U_s is changing stepwise. The first step is equal to half of the maximal value, every next step is equal to half of the previous. In this way every step represents subsequent bit (in two-digit code), starting from the most significant bit. The standard voltage is closing to the measured voltage in successive approximation. After every step the comparator sends the signal to output – this signal is *one* if measured voltage is larger than the standard voltage. If standard voltage exceeds measured voltage on the output is send *zero* signal and this step is canceled. As result at the output we obtain the zero-one sequence representing in digital way the analog measured voltage.

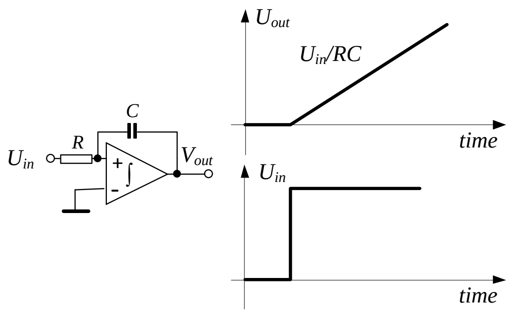

FIGURE 2.6
The integrating amplifier as the source of linearly increasing voltage.

Instead of stepwise increased voltage to realize digital compensation device we can use linearly increased voltage. As the source of such voltage can be used the integrating amplifier as it is explained in Figure 2.6.

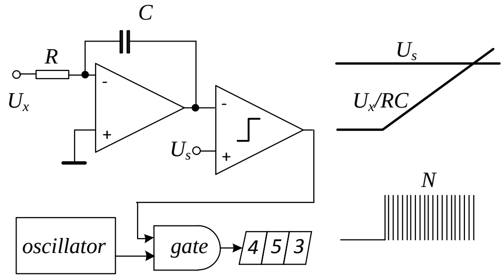

FIGURE 2.7
The integrating compensation device.

Figure 2.7 presents other analog-to digital converter basing on the compensation principle[4]. Linearly increased voltage is compared with standard voltage V_s. When output signal of integrating amplifier starts increasing then the gate is opened and pulses of oscillator are counted. Next when both voltages have the same values the comparator closes the gate. The number of pulses is proportional to measured signal.

We can compensate two signals (active values) – voltages, currents, magnetic fluxes. Easier is to compensate DC signals but also AC signals can be compensated (in this case two equilibrium conditions should be fulfilled – amplitude and phase equilibrium). But we are not able to compensate two passive values as for example resistance or impedance. In such case instead of subtraction X-X_s we can determine ratio X/X_s between these values.

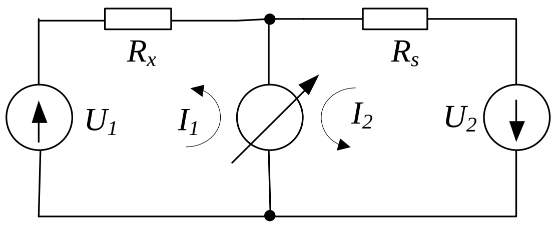

FIGURE 2.8
The principle comparator device.

An example of comparator device is presented in Figure 2.8. In the circuit presented in Fig. 2.3 we can obtain the equilibrium by the compensation of the currents I_1 and I_2

$$I_1 - I_2 = 0 \tag{2.7}$$

This state of equilibrium can be realized by the change of the voltage U_1 or U_2 . The condition of the equilibrium is

$$\frac{R_x}{R_s} = \frac{U_1}{U_2} \tag{2.8}$$

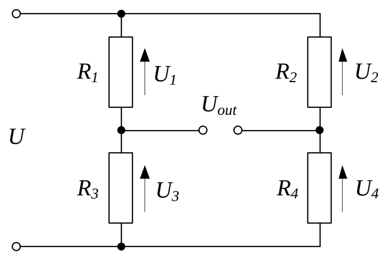

FIGURE 2.9
The principle of the bridge device.

Figure 2.9 presents other measuring device based on comparator principle. It is well known *bridge circuit*. We have connected parallel two voltage dividers R_1/R_3 and R_2/R_4. The balance condition (zero voltage on null indicator) is when $U_3 = U_4$. These voltages are equal to:

$$U_3 = U\frac{R_3}{R_1 + R_3}, \quad U_4 = U\frac{R_4}{R_2 + R_4} \tag{2.9}$$

Thus we can easy obtain condition of equilibrium:

$$R_3\left(R_2 + R_4\right) = R_4\left(R_1 + R_3\right) \tag{2.10}$$

and next:

$$R_3R_2 = R_1R_4 \tag{2.11}$$

Assuming that resistance R_1 is a measured resistance R_x we obtain the relation:

$$R_x = R_2\frac{R_3}{R_4} \tag{2.12}$$

If we fix the resistance ratio R_3/R_4 we can determine measured resistance directly from adjusted resistance R_2 (by changing the ratio R_3/R_4 we can change the range of our measuring instrument).

Balanced bridge circuit (with null indicator) was a long time ago very important method of measurement resistance and impedance (thus also capacity or inductivity). It was possible to obtain high accuracy

[4] Modified converted of such type is known as dual-slope converter.

because result of measurement was dependent only on the very accurate adjustable resistor and did not depend on the supplying voltage *U*. But recently we have very stable and accurate electronic current source I_s and it is much easier to determine resistance from the Ohm's law:

$$R_x = \frac{U_x}{I_s} \tag{2.13}$$

But the modified bridge circuit (known as *unbalanced bridge circuit*) is still very important measuring device converting change of resistance to voltage[5].

If we use *balanced bridge circuit* for measurement of resistance the accuracy of determination of resistance δR_x according to relation (2.12) depends on the accuracy of all rest resistors

$$\delta R_x = \sqrt{(\delta R_2)^2 + (\delta R_3)^2 + (\delta R_4)^2} \qquad (2.14)^6$$

We can easy improve significantly accuracy of this measurement by applying the *substitution method*. Let us assume that we measure resistance in two steps. First we connect to bridge circuit measured resistance R_x. Next, in the second step we substitute resistance R_x by standard resistance R_s. By changing this resistance we try to obtain the same result (for example equilibrium, but also unbalanced state is acceptable). It is obvious that in such case accuracy of the bridge resistors is not important and accuracy of measurement depends only on the accuracy of substituted standard resistor:

$$\delta R_x = \delta R_s \tag{2.15}$$

Other example of the substitution method is presented in Figure 2.10. It is very difficult to measure alternating current especially if it is distorted and of high frequency. In contrary we are able to measure DC current with very high accuracy. Therefore we perform measurement in two steps. As first we connect alternating current to the thin wire and we detect temperature of this wire (as result of heating by current). In the next step we substitute AC current by standard DC current and we change this current to obtain the same temperature of the wire. If both temperatures are the same it means that "effective" value of both current is the same and it is sufficient to determine only DC current.

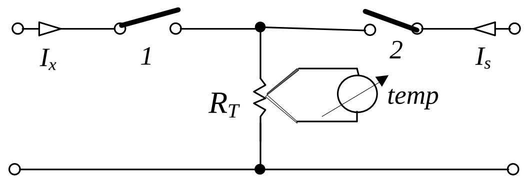

FIGURE 2.10
The example of the substitution method.

Pure comparison with standard value (by subtraction, by comparator or by substitution) is only one element of typical measuring system. Usually more often the measuring device is composed of many parts in form of a chain of *transducers* (converters). Every component can introduce errors and often the most weaken chain link can force the performances of the whole device.

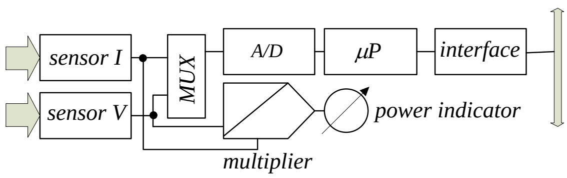

FIGURE 2.11
The example of the power measuring device.

Figure 2.11 presents an example of the device for power measurement. At first stage we should measure current and voltage. Next we can determine power by using an analogue multiplier. But we can also switch every value by multiplexer and next convert to digital values. By using microcontroller (or computer) we can perform more sophisticated operations, as calculation cosφ, reactive power, total harmonic distortion THD etc. Next we can send the final values by user interface to screen or printer but also we can transmit data by computer net or wireless transmitter. Thus *conversion* of measured values is very important and mostly used operation in measuring systems.

2.2 The conversion of measured values

Figure 2.12 presents selected example of the conversion devices. First of all there are a huge number of *sensors* converting various physical values into electrical value [Fraden 2003]. If as an output is a signal (voltage, current, etc.) we say that these sensors are *active sensors* and output signal can be transmitted

[5] Described later in chapter devoted to bridge circuits

[6] The relation (2.11) is explained later in chapter 2.5 - devoted to uncertainty of measurements.

to other devices. Often the measured value is converted into such parameter, as resistance, capacity etc. In this case, at the output of passive sensors we should connect the conditioning circuit converting this value into signal. Generally *conditioning circuit* [Pallas Areny 2001] beside conversion $\Delta R/R \rightarrow U$ includes also other functions as signal, amplification, errors correction, mathematical operation, even Ethernet, USB or wireless interface. Sometimes these circuits can be included into sensor – in this case we say about *intelligent sensors* [Manabendra Bhuyan 2011].

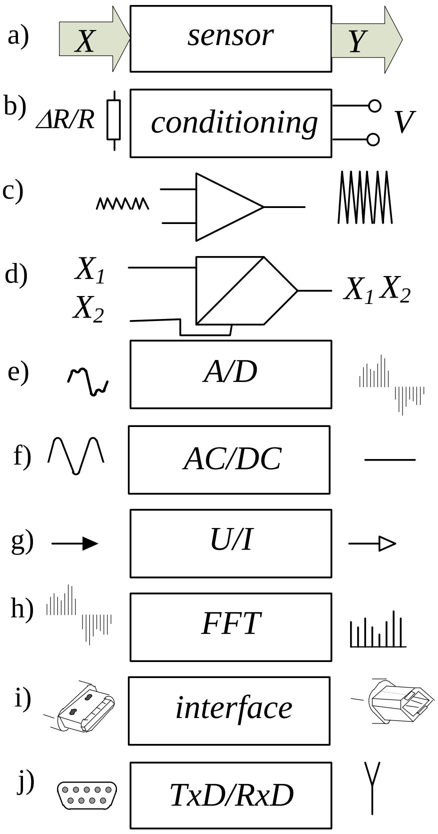

FIGURE 2.12
Various examples of the conversion of measured values: a) sensor, b) conditioning circuit, c) amplifier, d) mathematical operation – multiplier, e) analog to digital conversion, f) AC to DC conversion, g) voltage to current conversion, h) conversion from time domain to frequency domain, i) interface, j) transmitter - receiver.

We can convert analogue signal into digital (*A/D converter*) and next perform mathematical operation by using for example microcontroller. But often we can use analogue *mathematical converters*, as multiplier, integrating amplifier etc.

If we transmit analogue signal by wire it is convenient if the output signal is a current, because in this way change of resistance of the wire (caused for example by the changes of temperature) does not influence the result. Therefor for such purposes we use *voltage-to-current transducers*.

Signal can be processed in *time domain* but sometimes it is convenient if it is in *frequency domain*. Conversion between these domains is possible by using *Fourier transform*, for example *Fast Fourier Transform FFT*.

When we transmit signal to other device, for example to computer usually is used standardized connection known as *interface*. This connection can by realized by wire or wireless.

The data converters (transducers) [Kester 2005, Maloberti 2007] are described in more details in next chapters of this book. If they are used as measuring devices that should be described by their typical specifications as:

- accuracy,
- range (Full Scale FS),
- resolution,
- sensitivity (transfer function),
- linearity,
- influence of temperature (environmental factors),
- hysteresis and repeatability,
- crossfield effect,
- dynamic characteristic,
- excitation (power consumption)[7].

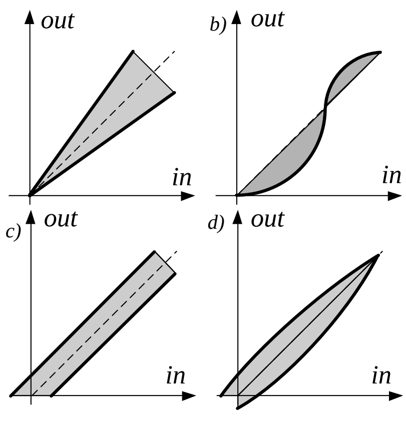

FIGURE 2.13
Typical errors of conversion: he example of the power measuring device: a) error of sensitivity, b) error of nonlinearity, c) error of resolution, d) hysteresis.

Performances of the measuring device are described by its dependence between output signal and input signal known as *transfer function*

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[7] Comprehensive review of such specifications is presented by [van Putten 1996]

$$out = K \cdot in \tag{2.16}$$

with transfer coefficient *K*.

The transfer coefficient is established during *calibration* of the device – it can be for example testing with measuring device of much better accuracy, assumed as standard or *reference device*. Transfer coefficient means practically the same as *sensitivity* $S = out/in$ but taking into account possible nonlinearity often we used differential sensitivity

$$S = \frac{\Delta out}{\Delta in} \tag{2.17}$$

The best is if we can describe transfer characteristic in form of mathematical relation. For example change of the resistance versus temperature of platinum thermoresistive sensor is:

$$\frac{\Delta R}{R_0} = At + Bt^2 \cong At \tag{2.18}$$

Similarly change of the resistance versus relative value of magnetic field h_x of magnetoresistive sensors [Tumanski 2000] is

$$\frac{\Delta R}{R_0} = \pm\frac{\Delta\rho}{\rho} h_x \sqrt{1-h_x^2} \cong \pm\frac{\Delta\rho}{\rho} h_x \tag{2.19}$$

where h_x is the value of magnetic field H_x related to anisotropy field H_k (H_k and $\Delta\rho/\rho$ - material parameters).

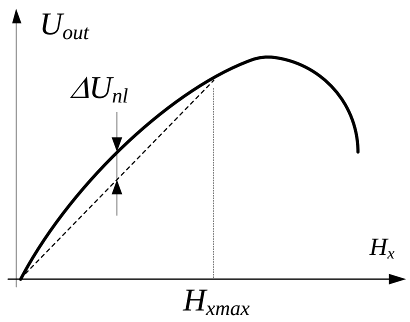


FIGURE 2.14
An example of transfer characteristic of magnetoresistive sensor (ΔV_{nl} – error of nonlinearity).

Figure 2.14 presents an example of the transfer characteristic described by relation (2.19). We see that often transfer characteristic is nonlinear and instead of the relation (2.16) we should use following equation:

$$out' = K(in) \cdot in \tag{2.20}$$

The error of non-linearity is

$$\delta_{nl} = \frac{out - out'}{out} = 1 - \frac{K(in)}{K} \tag{2.21}$$

As it is presented in Figure (2.14) we can decrease the nonlinearity to acceptable value by decreasing the range of input value. Thus nonlinearity (often in form of saturation for large value) limits the range of measuring device.

Sometimes we can obtain the effect of linearization by appropriate design of the transducer. For example sensitivity coefficient of a Hall sensor of thickness *t* is [Popovic 2004]

$$S = G_H \frac{R_H}{t} \tag{2.22}$$

Both the geometrical factor G_H and the Hall effect coefficient R_H depend on the carrier mobility μ_H but also on the measured magnetic field *B*

$$R_H = R_{HO}\left(1 - \alpha\mu_H^2 B^2\right) \text{ and } G_H = G_{H0}\left(1 - \beta\mu_H^2 B^2\right) \tag{2.23}$$

Fortunately the coefficients α and β have opposite signs and therefore it is possible to design a Hall sensor with a transfer characteristic close to linear. The best method of linearization is to use feedback because sensor works then only as zero detector (thus with very limited value of input signal).

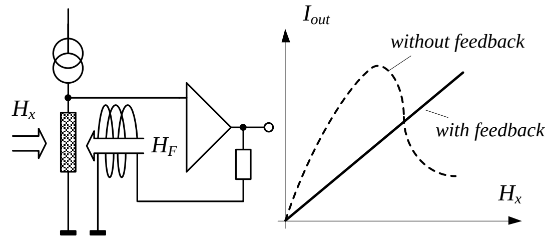


FIGURE 2.15
Linearization of transfer characteristic of magnetoresistive sensor by applying of a feedback.

Very important factor limiting performance of measuring device is resolution (Figure 2.13c). Usually the main source of limitation of resolution is noise. The main sources of noise are internal, for example resistance of the sensor is the source of *thermal Johnson noise* U_{nT} whilst semiconductor junction is the source of *shot noise* I_{ns}

$$U_{nT} = \sqrt{4kTR\Delta f} \tag{2.24}$$

$$I_{ns} = \sqrt{2qI\Delta f} \tag{2.25}$$

where k is the Boltzman constant and q is the electron charge.

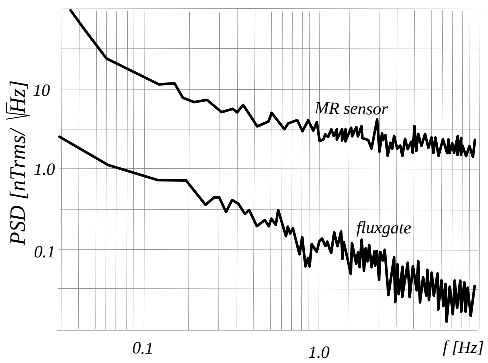


FIGURE 2.16
The example of power spectral density characteristic of noise determined for two types of magnetic sensors.

The noises can be described by the *power spectral density PSD of noise* (Figure 2.16). Because the noise depends on the frequency range Δf usually the spectral density $S(f)$ of noises is presented in form:

$$S(f) = \frac{U_n^2}{\Delta f} = \left(\frac{U_n}{\sqrt{\Delta f}}\right)^2 \tag{2.26}$$

Hence a "unit" of noise can be described for example as $\mu V / \sqrt{Hz}$ or a noise equivalent of measured value, for example in the case of magnetic field as $nT / \sqrt{Hz}$. Because level of noises depends on the frequency bandwidth Δf the best method to limit the noise is decrease of the frequency bandwidth – by using filters, selective amplifier of lock-in amplifier.

The second important limitation of the resolution is offset, in particular a temperature zero drift. Such problem is especially difficult in the case of resistive sensors where changes of resistance caused by measured value and changes of resistance caused by the temperature are not easy to separate.

Often the source of temperature zero drift lies in technology – defects in structure causing differences in heating of different parts of circuit. To decrease this effect special laser trimming technology can be used. The effective way to remove zero drift is a differential principle (described in the next chapter) or auto-zero function.

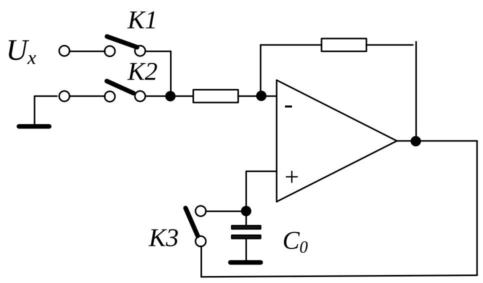


FIGURE 2.17
The example of auto-zero function.

Figure 2.17 presents the principle of temperature zero rejection by auto-zero function. In the first step switches K2 and K3 are closed. The amplifier detects its own zero drift. This drift is saved on the capacitor C_0. In the next step switches K2, K3 are disconnected while switch K1 is connected. The saved on capacitor zero signal is now subtracted from measured signal.

Practically almost all measuring devices are influenced by temperature. By an appropriate design it is possible to prepare temperature compensated sensors. For example temperature error of magnetoresistive sensor depends on the temperature changes of magnetoresistitvity $\alpha_{\Delta\rho}$ and the temperature changes of anisotropy α_{Hk}

$$\alpha_t = \alpha_{\Delta\rho} - \frac{H_k}{H_k + H_y + tM / w}\alpha_{Hk} \tag{2.27}$$

For Permalloy $\alpha_{\Delta\rho} \approx -0.018\ K^{-1}$ and $\alpha_{Hk} \approx -0.022\ K^{-1}$ and by appropriate design of thickness t and width w of the magnetoresistive strip we can obtain temperature self-compensating sensor.

For vector measurement important can be *crossfield effect*. It means that although we detect one component of measured value but second, orthogonal one influence the result.

In analysis of the response of measuring circuit we often neglect dynamic (time) effect assuming that we have steady conditions. But sometimes process of reaching steady value can be longer than time when we performed measurement. If we performed measurement too early we can made significant error as it is indicated in Figure 1.18b. Dynamics of transducers is very important for control technique especially that exist sensors of chemical values with very poor dynamic conditions. For example detectors of oil pollution sometimes need several minutes to obtain stable condition what is not acceptable for monitoring alert systems.

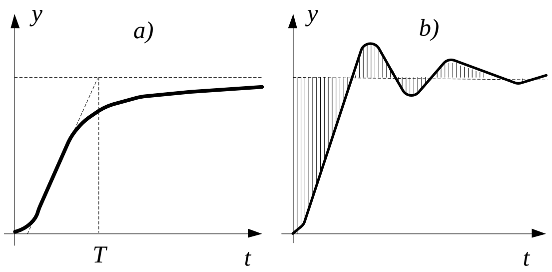


FIGURE 2.18
The response to the step input of device with inertia (a) and with oscillation (b). Dashed area – error caused by too early reading.

There are two methods of analysis of dynamic properties – versus time and versus frequency. In time analysis we introduce a stepwise change of input value and observe answer versus time. The most often answer is with inertia as is illustrated in Fig. 2.18a. The inertia type circuit can be characterized by the *time constant T*. For the first order inertia this time constant can be determined as the $0.638{\cdot}y_F$ (y_F – final value) or by drawing a tangent line to the response curve.

Inertial response $y(t)$ for input signal $x(t)$ is typical for *first order transducer* described by equation:

$$\frac{dy}{dt}T + y = kx \tag{2.28}$$

where T is time constant and k is a static transfer coeffici8ent.

For stepwise input $x(t) = A\,\mathbf{1}(t)$ the answer is described as:

$$y(t) = kA\left(1 - e^{-t/T}\right) \quad \text{or} \quad Y(s) = \frac{kA}{1+sT}X(s) \tag{2.29}$$

In the case of *second order transducer* described by the equation:

$$\frac{d^2y}{dt^2} + 2b\,\omega_0\frac{dy}{dt} + \omega_0 y = \omega_0^2 kx \tag{2.30}$$

where b is a damping coefficient and ω_0 is a resonance frequency.

the answer can be inertial or oscillatory depending on the damping. Transition between inertial and oscillatory answer is for $b = 1$. The equations describing answer for the stepwise input are more complex:

$$y(t) = y_u\left[1 - \frac{1}{\sqrt{b^2-1}}e^{-b\omega_0 t}sh\left(\sqrt{b^2-1}\omega_0 t + arth\frac{\sqrt{b^2-1}}{b}\right)\right] \tag{2.31}$$

in the case of output with inertia or

$$y(t) = y_u\left[1 - \frac{1}{\sqrt{1-b^2}}e^{-b\omega_0 t}\,sin\left(\sqrt{1-b^2}\,\omega_0 t + arctg\,\frac{\sqrt{1-b^2}}{b}\right)\right] \tag{2.32}$$

in the case of oscillations.

Therefore the second order transducer is more convenient describe versus frequency by the transform:

$$G(s) = \frac{Y(s)}{X(s)} = \frac{K\omega_0^2}{s^2 + 2b\omega_0 s + \omega_0^2} \tag{2.33}$$

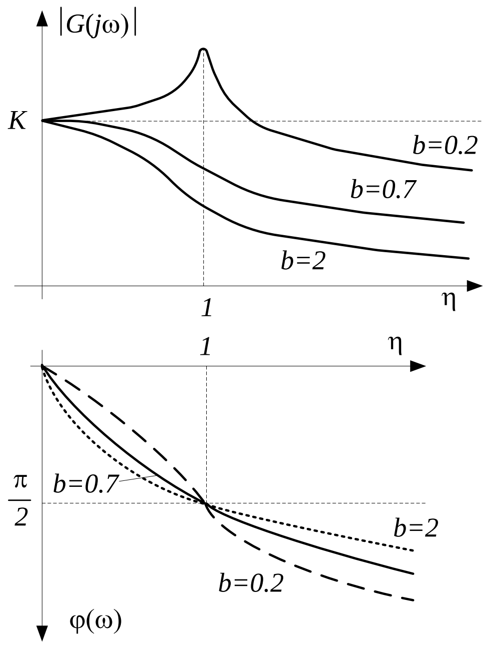


FIGURE 2.19
The amplitude and phase characteristics of the second order transducer.

The amplitude and phase characteristics (Figure 2.19) are described by:

$$|G(j\omega)| = \frac{K}{\sqrt{(1-\eta^2)^2 + (2b\eta)^2}} \qquad (2.34)$$

$$\varphi(\omega) = arctg\,\frac{2b\eta}{\eta^2 - 1} \qquad (2.35)$$

where $\eta = \omega / \omega_0$.

The device processes the dynamic signal without distortion if the amplitude is constant with frequency:

$$|G(j\omega)| = const \qquad (2.36)$$

Important also is the phase condition in the form:

$$\varphi(\omega) = 0 \;\; or \;\; \pi \;\; or \;\; k\omega \qquad (2.37)$$

From the characteristics presented in Figure 2.19 we can see that transducer can be used for plateau of amplitude characteristic - for $\omega < \omega_0$. It can be proved [Hagel, Zakrzewski 1984] that optimal value of damping is *b = 0.707* when the plateau is the widest and phase characteristic is close to linear.

2.3 Feedback and differential operation in measuring systems

In the measuring systems the feedback is very advantageous and it should be applied always if it is possible. Let us compare the performances of open-loop and feedback voltage transducers – presented in Figure 2.20.

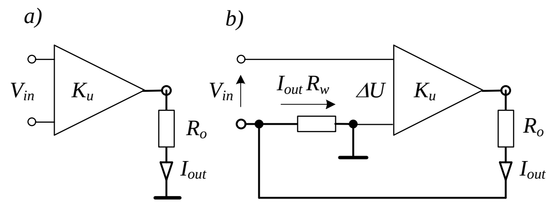


FIGURE 2.20
The voltage transducer: without feedback (a) and with feedback (b).

If the transducer operates without feedback (Fig. 2.20a) its conversion factor is

$$K' = \frac{I_{out}}{U_{in}} = K_u \frac{1}{R_o} \qquad (2.38)$$

Thus this factor directly depends on the gain factor of the amplifier. Usually, it is rather difficult to ensure stable gain, which is varying with the temperature, supply voltage or by the aging of the elements. If we apply the current feedback (Fig. 2.20b) then the conversion factor is

$$K = \frac{K_u}{1 + K_u\beta} = \frac{1}{\frac{1}{K_u} + \beta} \approx \frac{1}{\beta} \qquad (2.39)$$

where β is the feedback coefficient.

We see that gain factor does not influence the result and transfer coefficient depends only on the feedback (if gain is very large what usually is fulfilled). In our case the feedback coefficient depends only on resistance R_w – we can easy prepare this resistance as stable and with high accuracy.
After differentiation of (2.39) we obtain:

$$\frac{dK}{K} = \frac{1}{1 + K_u\beta}\frac{dK_u}{K_u} - \frac{K_u\beta}{1 + K_u\beta}\frac{d\beta}{\beta} \qquad (2.40)$$

Usually the feedback elements are stable and precise (in our example it is the resistance R_w), thus we can assume $d\beta/\beta \cong 0$. The equation (2.40) is:

$$\frac{dK}{K} \cong \frac{1}{1 + K_u\beta}\frac{dK_u}{K_u} \qquad (2.41)$$

As larger factor $K_u\beta$ (depths of feedback) as more negligible are changes of the gain of the amplifier.

Thus after application of the feedback *the accuracy of the transducer* increases significantly. It should be noted that the feedback decreases only multiplicative errors, the additive errors (for example zero drift) do not decrease with feedback.

The feedback improves also the *linearity of the transducer*. The input signal of the amplifier is

$$\Delta x = x_{in} - \beta\, y_{out} \qquad (2.42)$$

and because

$$y_{out} = K_u \Delta x \qquad (2.43)$$

the input signal of the amplifier is decreased by $(1+K_u\beta)$

$$\Delta x = \frac{x_{in}}{1 + K_u\beta} \qquad (2.44)$$

One of the sources of the nonlinearity is large range of input voltage of the amplifier (close to the saturation). If the input signal is small we use only linear part of the amplifier transfer characteristic. For the circuit presented in Fig. 4.20 the equations (2.42 – 2.44) are:

$$\Delta V = V_{in} - I_{out} R_w ; \quad I_{out} = \Delta V K_u \frac{1}{R_o + R_w} \tag{2.45}$$

$$\Delta V = \frac{V_{in}}{1 + K_u \dfrac{R_w}{R_w + R_o}} \tag{2.46}$$

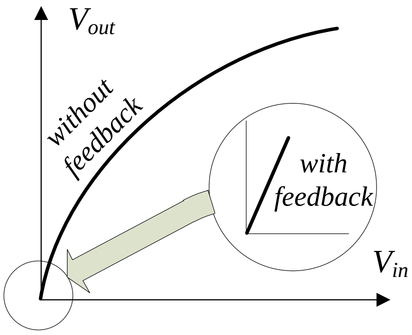


FIGURE 2.21
By applying the feedback we use only small linear part of the whole characteristic.

The input signal of the amplifier ΔV is significantly smaller than the input signal V_{in} of the whole transducer (for example if we process an input signal in the range of *mV* the input signal of the amplifier is in the range of μV). It means that we use only small linear part of the characteristic of amplifier (Figure 2.21). As it is presented in Figure 2.15 feedback improves also linearity of the nonlinear sensor.

It is recommendable if the transducer exhibits large input resistance, because the source of the signal is not loaded. Moreover, if the resistance of the source R_s is varying it does not influence the accuracy. The feedback enables significant increase of the input resistance. For the transducer presented in Fig. 4.20b we can write that

$$I_{in} = \frac{V_{in} - I_{out} R_w}{R_{in} + R_w + R_s} \tag{2.47}$$

Taking into account the dependencies (2.44) we obtain

$$I_{in} = \frac{V_{in}}{R_{in} + R_w + R_s} \frac{1}{1 + K_u \beta} \tag{2.48}$$

Without the feedback (Fig. 2.20a) we have

$$I_{ino} = \frac{V_{in}}{R_{in} + R_s} \tag{2.49}$$

Neglecting the resistance R_w as rather small we can state that after applying of the feedback the input current decreases by factor of ($1+K_u\beta$) and

$$R_{in} = (1 + G\beta) R_{ino} \tag{2.50}$$

where R_{ino} is the input impedance without feedback.

Similarly, we can prove that the output impedance of the transducer with current feedback is

$$R_{out} = R_{outo} + R_w (1 + K_u) \tag{2.51}$$

while the output impedance of the transducer with voltage feedback is

$$R_{out} = \frac{R_{outo}}{1 + K_u \beta} \tag{2.52}$$

By applying the current feedback we obtain the transducer with current output (large resistance – current source). By applying of the voltage feedback we obtain the transducer with voltage output (small resistance – voltage source).

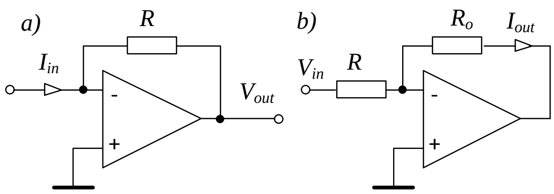


FIGURE 2.22
Current to voltage (a) and voltage to current (b) converters.

Figure 2.22 presents two converters where due the feedback it is possible to arrange input and output resistances. In current to voltage converter (small output resistance) the output signal is:

$$V_{out} = -R I_{in} \tag{2.53}$$

while in reverse converter (large output resistance) the output current is:

$$I_{out} = \frac{V_{in}}{R} \tag{2.54}$$

Feedback helps also in improvement of the dynamic performances of the transducer. If the open circuit is inertial and is described by the following transmittance

$$G(s) = \frac{K_u}{1 + sT} \tag{2.55}$$

then the transmittance of such circuit with feedback is

$$K(s) = \frac{G(s)}{1 + \beta G(s)} = \frac{K_u}{1 + \beta K_u} \frac{1}{1 + s\dfrac{T}{1 + \beta K_u}} \tag{2.56}$$

We see that the time constant *T* decreases by a factor of ($1+\beta Ku$) (the sensitivity also decreased by the ($1+\beta Ku$) factor). Fig. 2.23 presents the comparison of the response for the step function.

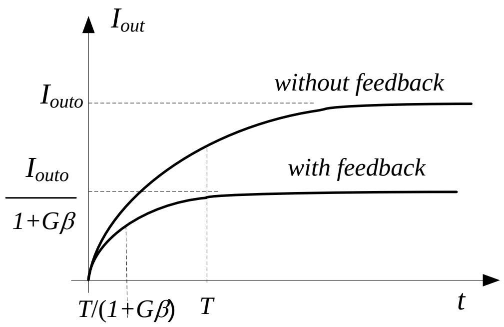


FIGURE 2.23
The response to the stepwise input of inertia transducer.

Also in the case of the oscillation type of the transducer we obtain improvement of the performance after applying the feedback. Without the feedback the transmittance is:

$$G(s) = \frac{K_u \omega_o^2}{\omega_o^2 + 2b\omega_o s + s^2} \tag{2.57}$$

where ω_o is the resonance frequency and *b* is the damping coefficient of the oscillations.

After applying of the feedback the transmittance is

$$K(s) = \frac{K_u \omega_o^2}{\left(\omega_o \sqrt{1+K_u\beta}\right)^2 + 2\left(\omega_o \sqrt{1+K_u\beta}\right)\left(\dfrac{b}{\sqrt{1+K_u\beta}}\right)s + s^2} \tag{2.58}$$

We see that with the feedback the resonance frequency increases by $\sqrt{1+K_u\beta}$ while damping decreases by a factor of $\sqrt{1+K_u\beta}$. The comparison of the frequency characteristics for the circuits with and without the feedback is presented in Fig. 2.24.

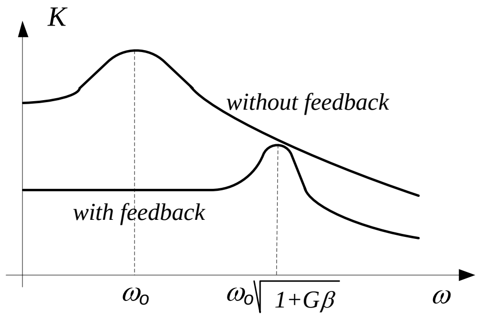


FIGURE 2.24
The frequency characteristic of the transducer of oscillatory type.

Feedback can be realized also by other than electrical way. Figure 2.25 presents the force transducer. Measured force F_x causes the deflection of the bar and moves the displacement sensor P_1 from the state of balance. The output signal of this sensor after amplification is connected to the coil of electromagnet P_2. The force of repulsion of this coil moves the bar back in order to obtain again the state of balance (and zero signal from the sensor P_1). Therefore this transducer is also called the current weight.

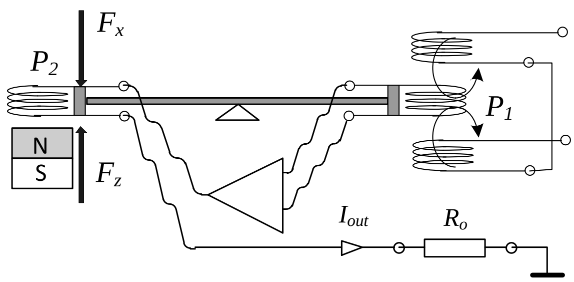


FIGURE 2.25
The transducer of force with feedback and current output.

The output current creates the balancing repulsion force

$$F_z = BzdlI_{out} = k_1 I_{out} \tag{2.59}$$

where B is the induction of the electromagnet, d an l are the dimensions of the coil and z is a number of turns.

Thus the output current is proportional to the measured force

$$I_{out} = kF_x \tag{2.60}$$

This transducer can be used for measurement pressure and flow.

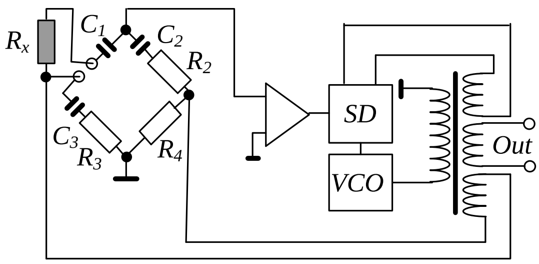


FIGURE 2.26
The transducer of resistance to frequency.

Figure 2.26 presents another type of transducer with feedback. This transducer converts the resistance to the frequency signal and the frequency is a feedback.

There is a certain group of bridge circuits, in which the condition of balance depends on the frequency of the supplying signal. For example, the balance condition for the bridge circuit presented in Fig. 2.26 is:

$$\omega = \sqrt{\frac{1}{C_2 C_3 (R_2 R_3 - R_4 R_x)}} \tag{2.61}$$

If we use the voltage controlled oscillator VCO the frequency is tuned to obtain balance of bridge.

We proved that feedback significantly improve performances of measuring transducer:

- improves of the accuracy,
- improves of the linearity,
- increases of input resistance (advantageous for voltage measurement),
- increases of output resistance (advantageous for signal transmission).

And what about the drawbacks? In some cases the circuit with feedback can be more complex. But the main drawback is the risk of instability – typical for all circuits with feedback. Fortunately we are able to design of appropriate correction PID circuits to assure the stable operation.

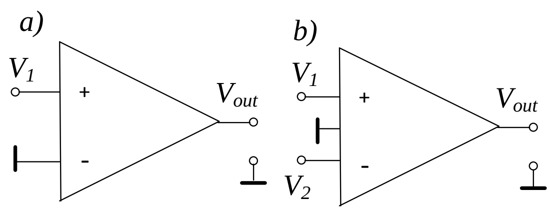


FIGURE 2.27
The single-ended (a) and differential (b) amplifier.

The feedback is very useful for decrease of multiplicative errors but it disappoints when exist additive errors, as for example temperature zero drift. In such case the differential operation can be helpful.

Figure 2.27b presents the the *differential amplifier*. The important advantage of such an amplifier is the possibility of suppression of the parasitic signals. The input signal is processed as the difference of two inputs signals

$$V_{out} = K_u (V_1 - V_2) \tag{2.62}$$

The parasitic interference signals ΔV are the same on both inputs. Therefore the output signal is

$$V_{out} = K_u \left[(V_1 + \Delta V) - (V_2 + \Delta V) \right] = K_u (V_1 - V_2) \tag{2.63}$$

Thus it is possible to amplify the voltage difference with the large common signal ΔV in the background. The possibility of the rejection of the common parasitic component is described by the coefficient *CMRR – Common Mode Rejection Ratio* defined as

$$CMRR = 20 \log \frac{K^-}{K^+} \tag{2.64}$$

where K^- is the amplification of the voltage difference and K^+ is the amplification of the common signal.
Taking into account this parameter the output voltage is

$$U_{out} = (U_1 - U_2) K^- \left[1 + \frac{1}{CMRR} \frac{\Delta U}{(U_1 - U_2)} \right] \tag{2.65}$$

The second component in the square brackets of the equation (2.65) describes the error caused by the presence of the common component.

If we connect to the input of differential amplifier the resistive sensor and the reference resistor of the same resistance R_{x0} as it is presented in Figure 2.28 we obtain rejection of the common component:

$$V_{out} = K_u I_w \left(R_{x0} + \Delta R_x - R_{x0}\right) = K_u I_w \Delta R_x \quad (2.66)$$

This way we reject common zero component and output signal is proportional only to the change of resistance. For example if we use Pt100 temperature sensor it has resistance in 0°C equal to 100Ω and the change of resistance about 3.9%/10°C. Thus if we measure temperature 0 - 10°C and use current 1 mA we have large steady component 100mV and small change of signal proportional to temperature 3.9 mV (103.9 mV). But in differential circuit (Figure 2.28a) we amplify only signal 3.9mV.

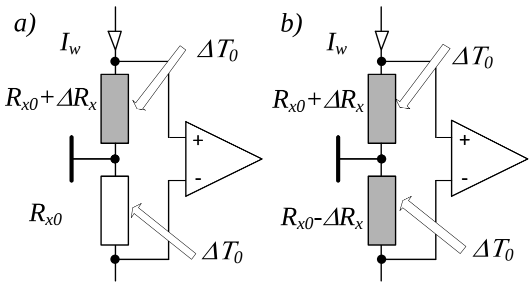


FIGURE 2.28
Differential connection of the resistive sensor: a) one active sensor, b) two differential sensors.

Much better results is possible to obtain if we connect two identical sensors – one active and second passive as the reference. For example in the Figure 2.29a we have two GMR magnetic sensors. One of them is active and the second one is passive (covered by shield). If the external temperature *T* changes the temperature zero drift is rejected:

$$R_{x0} + \Delta R_x\left(H_x\right) + \Delta R_x\left(T\right) - R_{x0} - \Delta R_x\left(T\right) = \Delta R_x\left(H_x\right) \quad (2.67)$$

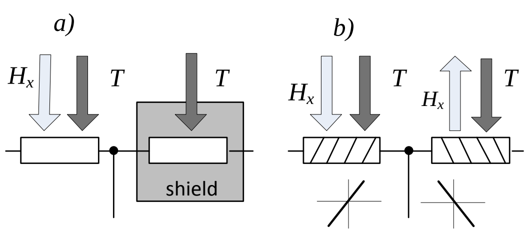


FIGURE 2.29
Passive and active magnetic field sensors (a) and two active differential magnetic field sensors (b).

We rejected both: steady zero component and temperature zero drift. Even better results we obtain if we have two differentials sensor. Differential sensors operate as follows:

$$R_1 = R_{x0} + \Delta R_x \text{ and } R_2 = R_{x0} - \Delta R_x \quad (2.68)$$

For example it is possible to design AMR magnetoresistive sensors that in one of them resistance increases and in the second one decreases versus measured magnetic field (Figure 2.29b) [Tumanski 2000]. In such case (Figure 2.28b) we reject both: steady zero component and temperature zero drift and the input signal is two times larger than in the case of one sensor.

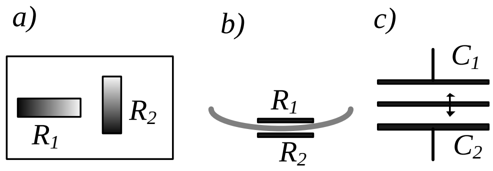


FIGURE 2.30
Passive and active differential sensors .

Fig. 2.30 presents other examples of the differential sensors. In the case presented in Fig. 2.30a two identical strain gauge sensors (sensors of mechanical strain or stress) are glued on the surface of stressed sample. But only one of these sensors (R_1) is stressed while the other (R_2) is placed perpendicularly to the stress. The temperature influences both sensors and as common component can be rejected.

Fig. 2.30b presents the stress measurement of the bended sample. The sensors R_1 is compressed, while at the same time the sensors R_2 is stretched. Similarly in capacitance differential sensor (Figure 2.30c) when internal electrode is moved one capacitance increases and second one decreases.

The most frequently as differential measuring circuit the bridge circuit is used (Figure 2.31). The output voltage depends on the changes of all four resistors as follows:

$$V_{out} \approx \frac{1}{4}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right)V_s \quad (2.30)$$

Thus, if the temperature influences two identical resistors R_1 and R_2 while the measured value influences the resistor R_1, then the output signal of the bridge circuit is:

$$V_{out} \approx \frac{1}{4}\left(\frac{\Delta R_1(x)}{R_1} + \frac{\Delta R_1(T)}{R_1} - \frac{\Delta R_2(T)}{R_2}\right)V_s = \frac{1}{4}\frac{\Delta R_1(x)}{R_1}V_s \quad (2.31)$$

and the influence of external temperature is eliminated. From Eq. (2.30) results that in bridge circuit we can use two pairs of differential sensors.

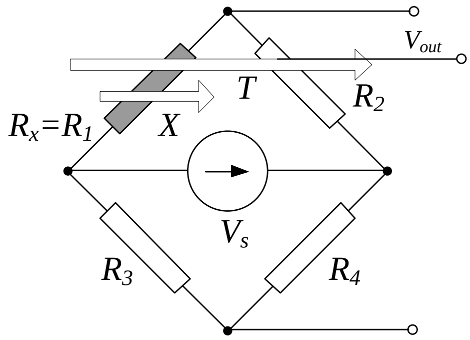


FIGURE 2.31
The bridge circuit a tool to apply the differential principle.

Basing on the idea presented in Figure 2.28 Anderson proposed measuring circuit known as Anderson loop (Figure 2.32) [Anderson 1994, 1998].

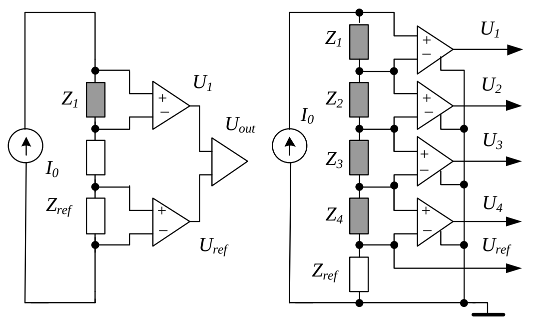


FIGURE 2.32
Two examples of Anderson loop.

In comparison with the bridge circuit the Anderson loop has two important advantages. In Anderson loop it is possible to connect simultaneously several sensors – the loop with four sensors is presented in Figure 2.32b. The output signal of each sensor can be determined as the difference between output voltage and reference voltage, for example

$$U_1 - U_{ref} = I_0\,\Delta Z_1 \quad (2.32)$$

Moreover the Anderson loop consumes smaller power than the bridge sensors what was important in NASA applications.

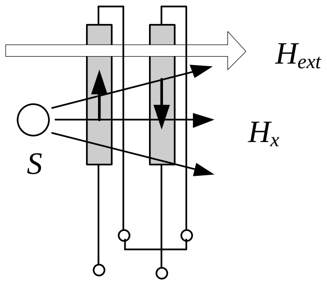


FIGURE 2.33
Two differential sensors used as a gradiometer.

By applying the differential operation we can reject also other parasitic signal. Fig. 2.33 presents a method of elimination of the influence of external magnetic field (for example Earth’s magnetic field) during the measurement of magnetic field from the source *S*. Such problem is common in biomedical measurements, when small magnetic field needs to be investigated, for example with magneto-cardiograph in presence of much larger Earth’s magnetic field. These two sensors are connected differentially and are positioned at some distance from each other. We can assume that the source of Earth’s magnetic field is large and it is at long distance from the sensors; therefore, the external magnetic field H_{ext} is the same in both sensors. The investigated source of magnetic field is small and near the sensors thus sensor placed closer to this source is influenced more than the other sensor positioned at some distance from the source *S*. Such pair of sensors is known as gradiometer device because this device detects the gradient of magnetic field.

2.4 Signal characteristics

The information obtained as the result of measurement is usually processed as a measurement signal. As the *measurement electric signal* we mean the time varying electric signal representing measured value. Various signal parameters can be used as the representation of the measured value: magnitude, frequency, phase, etc. Usually electric voltage (or current) with sufficiently large magnitude is preferred.

Recently commonly as the signal carrier the *digital signals* are used. We divide the signals into *analogue* and *digital* (*discrete time signals*) (Figure 2.34). In the case of analogue signal we usually know the value in every moment (*continuous time signals*) and in the case of *periodic signal* it is possible to describe it using the sinus function:

$$x(t) = X_m \sin 2\pi f \tag{2.33}$$

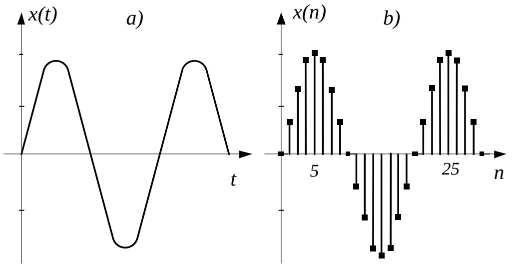


FIGURE 2.34
Analogue (continuous time) and digital (discrete time) signals.

A digital signal is obtained by determination of its value (usually in binary code) only in selected moments (*discrete time signal*). Instead of time it is described by number of the sample *n*:

$$x(n) = X_m \sin 2\pi fnT_s \tag{2.34}$$

where T_s is a period of sampling.

Most of physical phenomena are analogue and digital signals are slightly artificial, with their own mathematic tools. Therefore they are discussed separately in the chapter devoted to digital signal processing.

The signals can be deterministic or stochastic. The *deterministic signals* can be predicted with certainty and are reproducible. In the case of the *stochastic signals* we can only predict (estimate) them with some level of probability. We use tools of theory of probability to describe and analyze the stochastic signals.

The DC signal is described by one parameter – its value. The AC signal can be described by various parameters: the magnitude U_m or peak value U_p, mean value U_0, average (rectified) value U_{AV}, effective (*rms – root mean square*) value U_{rms}, peak-to-peak value U_{pp}, instantaneous value $u(t)$. Moreover, we should know the frequency f (or $\omega=2\pi f$ or period $T=1/f$) and the phase φ.

If the voltage signal is described by the equation (2.33) its main parameters are as follows

$$U_0 = \frac{1}{T}\int_{t_0}^{t_0+T} u(t)dt \tag{2.35a}$$

$$U_{AV} = \frac{1}{T}\int_{t_0}^{t_0+T} |u(t)|dt \tag{2.35b}$$

$$U_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} u^2(t)dt} \tag{2.35c}$$

It is easy to calculate that for sinusoidal signal these parameters are

$V_0 = 0$; $V_{AV} = 0{,}637V_m$; $V_{rms} = 0{,}707V_m$; $V_{pp} = 2U_m$

Even if AC signal is not pure sinusoid but it is periodic it can be described as the sum of harmonics (by applying the *Fourier Series*)

$$x(t) = a_0 + 2\sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t) \tag{2.36}$$

where: $a_0 = \frac{1}{T}\int_{t_1}^{t_1+T} x(t)dt$, $a_k = \frac{1}{T}\int_{t_1}^{t_1+T} x(t)\cos(k\omega_0 t)dt$,

$$b_k = \frac{1}{T}\int_{t_1}^{t_1+T} x(t)\sin(k\omega_0 t)dt$$

or in exponential form

$$x(t) = \sum_{k=-\infty}^{+\infty} c_n e^{jk\omega_0 t} \tag{2.37}$$

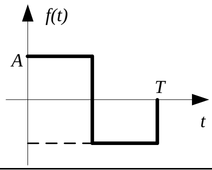
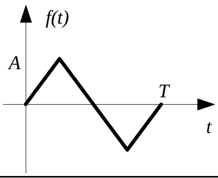
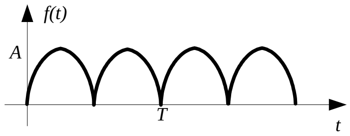
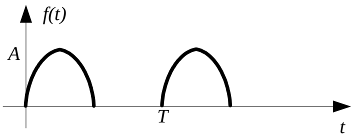
where: $c_n = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)e^{-jk\omega_0 t}dt$

When the function $x(t)$ is even (*in mathematical sense*) then coefficients $b_k = 0$ and when the function $x(t)$ is odd then $a_k = 0$. Table 2.1 presents the Fourier representation of some typical signals.

Deviation from the pure *sinusoidal waveform* is described by *total harmonic distortion THD* (as the percentage ratio of all harmonics components above the fundamental frequency to the magnitude of fundamental component):

$$THD = \frac{\sqrt{\sum_{k=2}^{n} V_k^2}}{V_1} \cdot 100\% \tag{2.38}$$

TABLE 2.1
Fourier representation of typical signals.

	$f(t) = \frac{4A}{\pi}\left[\sin(\omega_0 t) + \frac{1}{3}\sin(3\omega_0 t) + \frac{1}{5}\sin(5\omega_0 t) + \ldots\right]$
	$f(t) = \frac{8A}{\pi^2}\left[\sin(\omega_0 t) - \frac{1}{3^2}\sin(3\omega_0 t) + \frac{1}{5^2}\sin(5\omega_0 t) + \ldots\right]$
	$f(t) = \frac{2A}{\pi} - \frac{4A}{\pi}\sum_{n=1}^{\infty}\cos 2n\omega_0 t$
	$f(t) = \frac{A}{\pi} + \frac{A}{2}\sin\omega_0 t - \frac{2A}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2 - 1}\cos 2n\omega_0 t$

The distorted signal can be presented as a Fourier series also in a graphical form. Usually the signals are presented in form a line spectrum where the individual harmonics are represented by vertical lines (Fig. 2.35).

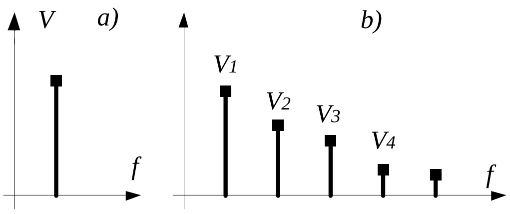


FIGURE 2.35
An example of the spectral analysis of the sinusoidal signal (a) and distorted signal (b).
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We see that the same signal can be presented in two forms – *in time domain* (Figure 2.34) or in *frequency domain* (Fig. 2.35) - (both methods are complementary). The conversion between signal described in time domain and frequency domain is possible using Fourier transform:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{j\omega t}d\omega \tag{2.39}$$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt \tag{2.40}$$

Indeed, as it illustrates Figure 2.36 time domain or frequency domain it is only other point of view on the same signal.

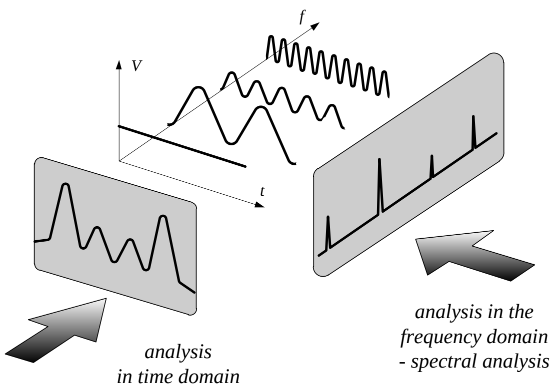


FIGURE 2.36
An example of the spectral analysis of the sinusoidal signal (a) and distorted signal (b).

More complex is the analysis of non-periodic signals because we cannot use the Fourier series rules. In this case instead of Fourier series (2.36) we can use Fourier integral transform (2.40) (by treating an aperiodic signal as a periodic with an infinite period). Mathematically we are able to analyze only simple waveforms. Figure 2.37 presents Fourier transform of rectangle pulse and *sinωt/t* signal.

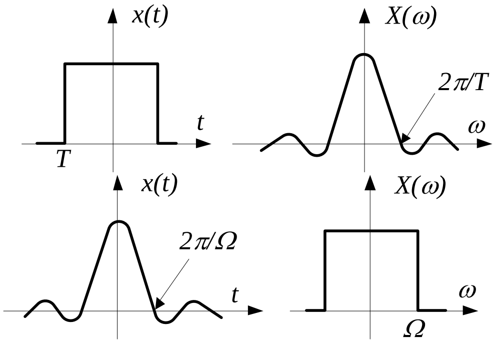


FIGURE 2.37
Fourier transform of rectangle pulse and *sinωt/t* signal.

If the signal in time domain is the rectangle pulse

$$x(t) = \begin{cases} 0 & for \quad |t| > T \\ 1 & for \quad |t| \le T \end{cases} \tag{2.41}$$

the Fourier transform is

$$X(\omega) = 2\frac{\sin \omega T}{\omega} \tag{2.42}$$

And reversely if signal in time domain is

$$x(t) = 2\frac{\sin \Omega t}{t} \tag{2.43}$$

the Fourier transform is a rectangle.

The Fourier transform is reversible – it means that we always can return to previous time domain signal by using inverse Fourier transform (2.39). But the signals should be *stationary* - are constant in their statistical parameters over time. In non-stationary signals we can test periodicity by using *autocorrelation function*:

$$R_{xx}(\tau) = \frac{1}{T}\int_{-\infty}^{\infty} x(t)x^*(t-\tau)dt \tag{2.44}$$

We multiply signal with it complex conjugate shifted by time τ - this way we test is exists similarity between this two parts of the signal. We can also test similarity of two various signals by co-corellation function:

$$R_{xy}(\tau) = \frac{1}{T}\int_{-\infty}^{\infty} x(t)y^*(t-\tau)dt \tag{2.45}$$

If we have stochastic signal we can test its mean value:

$$\overline{x} = \frac{1}{T}\int_0^T x(t)dt \tag{2.46}$$

variance (spread around mean value):

$$S_x^2 = \frac{1}{T}\int_0^T \left[x(t) - \overline{x}\right]^2 dt \tag{2.47}$$

standard deviation

$$S_x = \sqrt{\frac{1}{T}\int_0^T \left[x(t) - \overline{x}\right]^2 dt} \tag{2.48}$$

and rms value:

$$x_{rms} = \sqrt{\frac{1}{T}\int_0^T x(t)^2 dt} \tag{2.49}$$

2.5 Uncertainty of measurements

The International Organization of Standardization (ISO) with collaboration of many other prestigious organizations edited in 1993 a "*Guide to the expression of uncertainty in measurement*"- usually known as GUM[8]. This document was a result of thousands discussions in metrological milieu and many years of preparation. Today, we can say that before the *Guide* there was the theory of errors and after the *Guide* there is the theory of uncertainty in measurements.

Unfortunately the *Guide* did not solve the problem of understanding of measurement accuracy, because it is written with very difficult style and it is clear only for very narrow circle of specialists. No wonder that after the *Guide* the frustration of people active in measurements deepened and the milieu divided to the

[8] Recently valid is version JCGM 100:2008 "Evaluation of measurement data — Guide to the expression of uncertainty in measurement" developed by JCGM – Joint Committee for Guides in Metrology available in BIPM's website (www.bipm.org) - BIPM (The International Bureau of Weights and Measures (French: *Bureau international des poids et mesures*)

initiated peoples, who understand the *Guide*, and the rest, who don't. A lot of publications explaining the terms from the *Guide* have been published (Coleman et al 1999, Dunn 2010, Fornasini 2008, Gertsbakh 2003, Hughes et al 2010, Kirkup et al 2006, Pavese et al 2009, Wheeler et al 2004, Rabinowich 2005, Taylor 1996). The *Guide* is an official document, as well as standard and law, therefore everyone is obliged to try understand it and to comply with it.

We should start with attempt to order of many, sometimes excluding terms, as: uncertainty, error, precision, estimated value, true value, measurand etc. In this task helpful should be document known as VIM (Vocabulaire international de métrologie) [9]

According to this vocabulary VIM *the error of measurement* is the difference between measured value and the *true value*. Because we seldom know the true value therefore better is to substitute an error by the *uncertainty of measurement* - parameter characterizing the dispersion of the measured value around the *estimated value* (attributed to *measurand*). The measurand[10] means quantity intended to be measured. It can be other than measured value due to for example influence of measuring equipment into measured value (for example when we measure the voltage with voltmeter of too small value or if we measure the temperature with thermometer distorting the distribution of temperature). Thus we can say that uncertainty is an estimation of the error in measurement.

In vocabulary VIM the *accuracy*[11] is only the ability of the measuring system to provide a quantity value close to the *true value.* It is not a quantity and describes only quality of measuring device (more or less accurate measurement). Also in vocabulary the *precision of measurement* means only agreement between measured quantity value obtained by replicate measurement (thus meaning similar to repeatability).

And what about error? The VIM accepts the usage of term "*error*" if we know with sufficient accuracy the reference value (attributed to true value). For example if we perform calibration of measuring device by comparison with standard device we can say about error. Similarly if we determine the difference between straight line and nonlinear characteristic we can also say about error of nonlinearity.

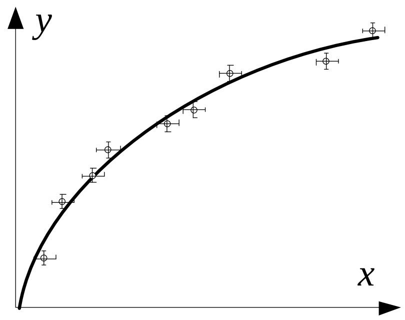


FIGURE 2.38
An example of graphical indication of uncertainty of measurement.

It is recommended to present the result of measurement with uncertainty of measurement – for example (5.255 ± 0.002) V or 5.255 V ± 0.01%, although VIM accepts results with not indicated uncertainty of measurement if it is negligible small. Figure 2.38 presents the example of graphical indication of uncertainty of results.

The error or uncertainty can be presented as *absolute error* (difference between results of measurement and mesurand X_M) or more convenient is to present it as *relative error* in % :

$$X = X_M \pm \Delta X \quad \text{or} \quad X = X_M \pm \frac{\Delta X}{X} X \qquad (2.50)$$

Unfortunately, the prevailing opinion (especially among students) is that the analysis of uncertainty is rather difficult and somewhat dull. Sometimes, people even say that the measurements would be interesting if not the theory of errors. On the other hand, if it is indispensable to use this theory better it is to grow fond of it. Moreover, in many cases the analysis of accuracy of measurement can be intellectually challenging and even can be more important and interesting than routine measurement procedure.

We can rewrite the equation (2.50) in the form representing an error:

$$X_T - \Delta X \le X \le X_T + \Delta X \qquad (2.51)$$

[9] "International vocabulary of metrology - Basic and general concepts and associated terms (VIM)" – document JCGM 200:2008 available in BIPM's website (www.bipm.org).

[10] The official documents of ISO consequently use the term measurand. For the sake of simplicity, and because the word measurand does not exist in Dictionaries of English, further in this book these parameters (measurand or value to be measured) are called "the measured value".

[11] In common talking, we can often come across a statement like: "the measurement was performed with the accuracy *0.1%*". It is of course logical mistake, because it means that the measurement was performed with inaccuracy *0.1%* (or accuracy *99.9%*). To avoid such ambiguity it is better to say "the measurement was performed with the uncertainty smaller than *0.1%*".

which can be read as follows: the result of measurement X is determined with the dispersion $\pm\Delta X$ around the true value X_T (bearing in mind that $\pm\Delta X$ is an absolute error of measurement).

According to the concept presented in the GUM the dependence (2.51) should be substituted by the dependence

$$\Pr(X_0 - u \le X \le X_0 + u) = 1 - \alpha \qquad (2.52)$$

which should be interpreted as follows: *the result of measurement X is determined with the uncertainty ± u around the estimated value X_0 with the level of confidence (1-α).* Symbol *Pr* in the equation (2.52) denotes the probability.

We can see that the true value (which we never know) is now substituted by *the estimated value*. Similarly, the error is now substituted by the *uncertainty*, because we also do not know the value of that error. Earlier the probability was attributed only to random errors. Now practically all uncertainties should be considered taking into account probability. Indeed if we measure voltage with digital instrument of high accuracy always we know the last digit as 0.5X <X<1.5 X with uniform probability (for example in result 2.255 all results between 2.2545 and 2.2555 are equally probable).

The resultant uncertainty of measurement can comprise several components: *corrections* (a), *random uncertainty* (b), *uncertainty related to the imperfect accuracy of measuring devices and methods* (c), *uncertainty related to non-perfect model of investigated phenomenon* (d) and *mistakes* (e).

a) The *correction* is the uncertainty ΔX_0 which we are able to determine and remove. For example the transfer characteristic of the thermoresistive sensor is often described by the dependence $R_T = R_0\,(1+\alpha T)$ – where R_0 is the resistance in temperature 0℃, R_T is the resistance in temperature T, α is the temperature coefficient. But more detailed analysis of the transfer characteristic leads to a conclusion that the thermoresitor is better described by the dependence: $R'_T = R_0\,(1+\alpha T+\beta T^2)$. Thus, if we do not take into account the nonlinearity of the sensor we make error of linearity:

$$\Delta R_{T0} = R'_T - R_T = R_0 \beta T^2 \qquad (2.53)$$

Because we know the value of this error of linearity we can remove it – for example by setting it into computer memory and subtracting it every time during measurement.

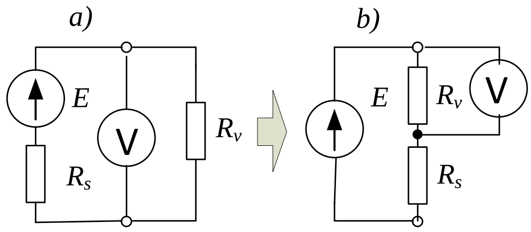


FIGURE 2.39
An error caused by voltmeter of finite internal resistance.

We can also take into account the correction when our measuring method exhibits error but we are able to determine it. For example, Figure 2.39a presents a voltmeter with internal resistance R_v connected to the source E with the internal resistance R_s. If we convert this circuit to the same presented in Figure 2.39b we see that both resistance create voltage divider and the voltmeter measures instead voltage E the voltage equal to E':

$$E' = \frac{R_v}{R_s + R_v} E \qquad (2.54)$$

Thus imperfect voltmeter introduces error $\delta E = (E' - E)/E$:

$$\delta V = \frac{R_s}{R_v + R_s} \qquad (2.55)$$

If resistance of the voltmeter is similar like resistance of source the voltmeter measures only half of voltage and error is about 50%. We can remove this error and correct the result of measurement if we know all the resistances (although better solution would be substitute this voltmeter by better one, if possible).

b) The *random uncertainty* exists when we can diminish it by increasing the number of measurements. If we perform several measurements and every time we obtain a slightly different result (with dispersion exceeding assumed value) we can conclude that the uncertainty is random. The uncertainty of measurement caused by the random character depends inversely on the number of measurements.

c) There are uncertainties, which we are able to estimate but we cannot remove. For example the voltmeter used previous example exhibits limited uncertainty described usually by the manufacturer (as the accuracy of scaling). The repeating of the measurements many times do not change this error. Generally this kind of uncertainty depends on the *uncertainty of used measuring devices* and can be estimated based on the information enclosed by the manufacturers of these instruments.

d) Another kind of uncertainties can result from the imperfect model of the investigated object or phenomenon. In the example described above (Figure 2.39) we removed the error introduced by the measuring method. But if the same circuit is supplied by the alternating current the model of the measuring circuit is much more complex than that presented by equation (2.54). In such case we should take into account the parasitic capacitances with respect to the ground, the capacitances and inductances of resistors, influence of frequency, influence of external electromagnetic field etc. The dependence (2.54) should be appropriately extended to include all these factors to the model.

Sometimes the model of phenomenon can be so complex that its application could be difficult, especially in industrial environment. In such case we can construct *artificial model* of the physical phenomenon as the result of the group agreement. For example, the magnetic parameter: “specific power losses” depends on a great number of factors – conditions of magnetization. It would not be reasonable to include all of them into the model. Therefore, the model of the losses has been limited to the precisely described one case – as this case the testing apparatus called the Epstein frame has been chosen. The Epstein frame has been very precise described in the international standard (EN 60404) – the method of preparation of the sample, the design of the apparatus, the measuring conditions, etc. have been established in details. The standardization of the measuring procedure guarantees that all investigators perform the same errorss and obtain comparable results in every laboratory. The results of these measurements are called “the Epstein losses”; in many cases being far from the real losses in the physical sense.

e) And last but not least, the worst kind of uncertainty – the unrecognized errors. These errors are sometimes called the *mistakes*. For example, we measure the current using damaged ammeter. Or we use this ammeter in the presence of magnetic field and we do not know that this field exists and influences the measurement. We have no reason to question the manufacturer declaration of accuracy and greater number of the measurement will not help. In such case only the *validation* of the measuring procedure (by means of the standard device called calibrator) could be effective but it is not always possible.

Figure 2.40 presents typical algorithm of evaluation of uncertainty. Thus the procedure of measurement should comprise the following steps:

- analysis and determination of the mathematical model of investigated object;
- analysis and determination of the mathematical model of the measuring system;
- analysis of source of errors and determination of the resultant uncertainty of measurement.

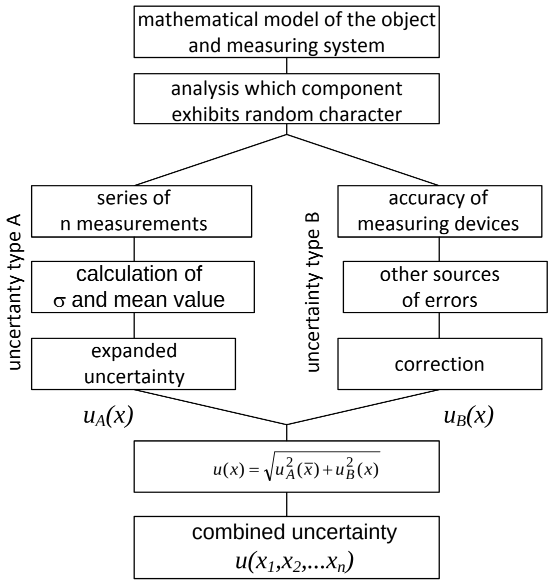


FIGURE 2.40
Algorithm of evaluation of uncertainty.

All points are crucial for correct assessment of uncertainty. The authors of the GUM recommend: “*Although this Guide provides a framework for assessing uncertainty, it cannot substitute for critical thinking, intellectual honesty and professional skill. The evaluation of uncertainty is neither a routine task nor a purely mathematical one; it depends on detailed knowledge of the nature of the measurand and of the measurement*....”.

Especially important is the first point. It should be emphasized that we always test the mathematical model of physical phenomenon or technological factor. As this model is closer to reality as better result of measurement and reversely if this model in not complete the measurement can be worthless. For example if we measure impedance for high frequency and do not include to our model skin effect we can obtain result far from true.

It is reasonable to limit the assumed uncertainty of measurement to the certain useful level. The increase of the accuracy means higher costs – we should use more expensive measuring devices, the time necessary for measurements is longer, the qualifications of the investigators should be higher. The application of inappropriate, too precise instruments can be the uncomfortable – for example when we use five-digit instrument for the measurement of non-stable source, the last digits keep blinking and are useless.

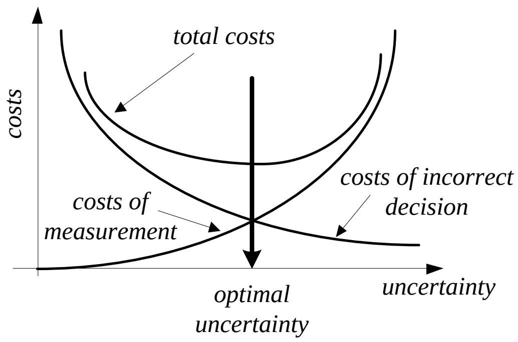


FIGURE 2.41
The dependence between the uncertainty and costs of measurements.

Figure 2.41 presents the graph illustrating the relation between the costs and uncertainty. We can reduce costs of measuring procedure by decreasing the uncertainty but it is the risk that the costs of incorrect decisions can be larger; they can even cause dangerous situations. Taking this into consideration we can establish the optimal value of uncertainty.

On the other hand recently even very accurate measuring devices are not expansive. Sometimes IC measuring device has performances reserved earlier for professional equipment. Therefore instead of time-consuming analysis easier is to take from the shelf better measuring instrument.

In some cases (health service, military industry, etc.) there is a need for increased accuracy. In this case the *Guide* proposes to substitute the *standard uncertainty u* by the *expanded uncertainty ku*. The *coverage factor k* is related to the level of confidence and typically is in the range *2 – 3*.

Returning to algorithm presented in Figure 2.40 the second step is the analysis which component exhibits random character. Before the analysis of uncertainty it is reasonable to execute the cycle of the measurements. If the dispersion of the results exceeds assumed value it means that we should perform statistical analysis of these results. In such case we use the procedure called by the *Guide* - ***type A evaluation of uncertainty.*** This kind of evaluation requires certain number of measurements – this number depends on the value of dispersion and the level of confidence.

But in certain circumstances it is not reasonable to repeat the same measurements many times. For example, we have stable supply sources, the conditions of environment are also stable (due to temperature conditioners, electromagnetic shield and grounding preventing the harmful interferences), we have very precise measuring devices. It would be just waste of time and money to repeat the measurements, especially in industrial environment. In such case we use the procedure described in the *Guide* - ***type B evaluation of uncertainty.***

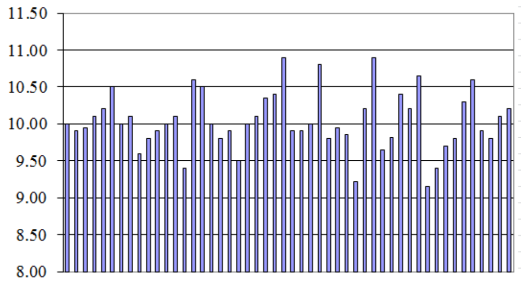


FIGURE 2.42
An example of the presentation of the measuring results in form a graph.

Consider the case when we perform a series of measurements and we obtain certain number of results in form of a table or a graph presented in Figure 2.42. We can easily analyze such set of results constructing *histogram*. The histogram can be calculated for instance with using simple tool in popular MS Excel program. Figure 2.43 shows the histogram of the data set presented in Figure 2.42.

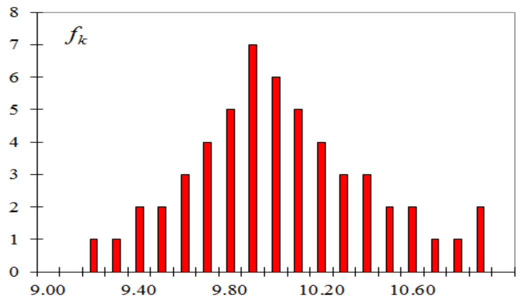


FIGURE 2.43
The histogram of the results of measurements presented in Fig. 2.42.

On the graph of histogram, the axis *x* describes the value of obtained result while the *y* axis presents value f_k describing how often such result happened. Analyzing the histogram we can obtain the information which value revealed most often – this value is probably the closest to the true value.

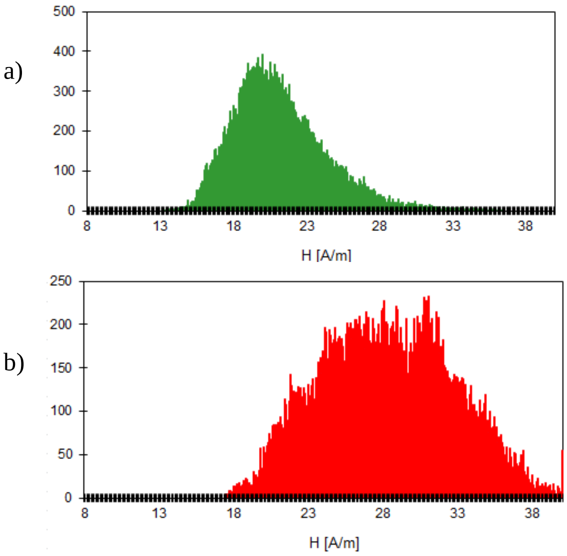


FIGURE 2.44
Two examples of the histograms of the test results of magnetic material homogeneity (Tumanski 1998).

Histogram is a very useful tool to analyze statistical data. Figure 2.44 presents examples of two histograms. They represent magnetic homogeneity of the material determined by scanning the magnetic field distribution of the selected area on investigated magnetic material (Tumanski 1998). We see that one of them (Figure 2.44a) is more uniform than the other one (Figure 2.44b), which is expressed by the slenderness of the histogram shape.

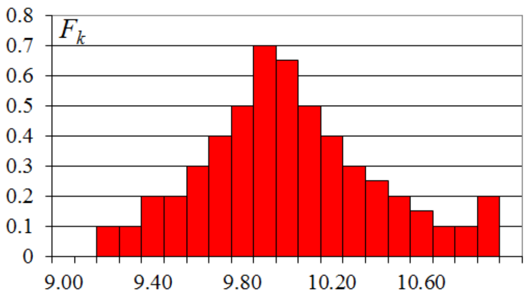


FIGURE 2.45
The histogram $f_k \Delta x$ of the results presented in Fig. 2.43.

It is also possible to calculate the histogram, in which the level of the bars is equal to the F_k value representing the area $f_k\,\Delta x$ (Figure 2.45). On the basis of such histogram we can evaluate in which Δx range the result of measurement happened most often. Analyzing the shape of the histogram (for example its width or slenderness) we can roughly estimate the uncertainty of measurements.

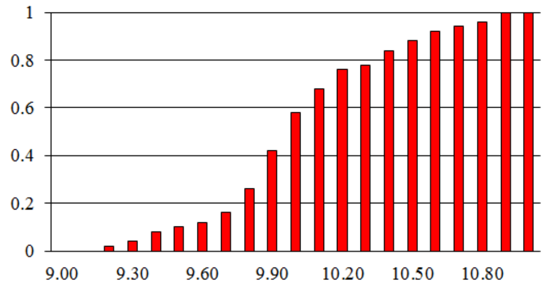
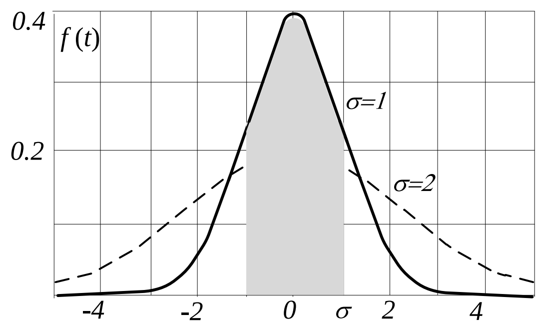


FIGURE 2.46
The cumulative histogram of the results presented in Fig. 2.43.

Another type of the histogram is presented in Figure 2.46. This cumulative histogram informs us how often happened the result in the range between -∞ and the *x* value (below 9 none result, till 11 is 100% of results).



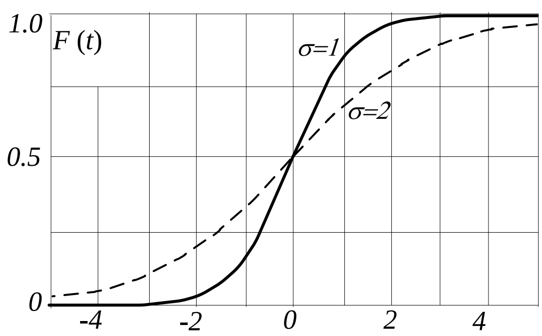


FIGURE 2.47
The graph of the density of probability *f(t)* and cumulative distribution *F(t)*for the normal distribution [12]

We can present the results of series of measurements as the probability distribution of the result instead of histogram. On the *y* axis we describe the probability density *f(x)* of the result of

[12] Figure 2.26 presents so called normalized functions of probability where $t = (x-\mu)/\sigma$

measurement. We can also calculate the cumulative distribution function *F(x)* as the area under the probability density function (Figure 2.47). The comparison of Figures 2.45, 2.46 and Figure 2.47 indicates that the probability density function is related to the histogram for infinite number of measurements (continuous function) while the cumulative distribution function is related to cumulative histogram. Very often the probability density function is represented by the *normal distribution* also called *Gaussian distribution*. The example of Gaussian distribution curves are presented in Figure 2.47.

The *distribution function* (cumulative distribution) describes the probability that the random variable be less or equal to x

$$F(x) = Pr(X \le x) \tag{2.56}$$

The *probability density of function* is the derivative of the distribution function

$$f(x) = \frac{dF(x)}{dx} \tag{2.57}$$

thus

$$f(x) = \Pr(x < X < x + dx)) \tag{2.58}$$

Knowing the probability density function we can determine the *probability* that the value X is in the range from x_1 to x_2

$$\Pr(x_1 < X < x_2) = \int_{x_1}^{x_2} f(x)dx \tag{2.59}$$

and of course is

$$\Pr(-\infty < X < \infty) = \int_{-\infty}^{\infty} f(x) = 1 \tag{2.60}$$

The *normal distribution* (Gaussian distribution) is described by the equation

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \tag{2.61}$$

thus it is described by two parameters: *standard deviation* σ and the *expected value* (expectation) μ.

The *expected value* is the value, around which all random variables are extended. For a continuous random variable having the probability density function *f(x)* the expected value μ is

$$\mu = \int_{-\infty}^{\infty} xf(x)dx \tag{2.62}$$

For the normal distribution the expected value is the symmetry axis of the function *f(x)* and for limited number n of observations x_i is equal to the *mean value* $\bar{x}$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{2.63}$$

We can determine the *standard deviation* σ as the positive square root of the *variance* $V(x) = \sigma^2$ given by equation

$$\sigma = \sqrt{V(x)} = \sqrt{\frac{1}{n-1}\sum(x_i - \bar{x})^2} \tag{2.64}$$

The variance *V(x)* describes the dispersion of the variable around the expected value:

$$V(x) = \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx \tag{2.65}.$$

The probability is equal to the area under probability density curve: for the σ value – grey area in Figure 2.47. For the normal distribution the probability that variable is equal to the expected value (in this case the mean value) with the dispersion equal to the standard deviation σ is

$$\Pr(\bar{x} - \sigma \le x \le \bar{x} + \sigma) = 0.6826 \tag{2.66}$$

Thus the probability that the result of observation is in the range $\pm\sigma$ around the mean value (expected value) is 68.26%. Similarly, we can calculate that this probability for the dispersion $\pm 2\sigma$ is 95.44% and for $\pm 3\sigma$ is 99.73%. We can say that the result of measurement is very close to estimated value if the uncertainty is 3σ. Therefore we sometimes say about *3σ rule* as the rule of large probability.

The standard deviation of the mean value depends on the number of observations n

$$\sigma(\bar{x}) = \frac{\sigma}{\sqrt{n}} \tag{2.67}$$

By increasing of the number of observations we diminish the range of uncertainty of the mean value. But the component $\sqrt{n}$ increases slowly with the increasing of n – to decrease the standard deviation by 10 it is necessary to increase the n by 100. Such effort (or waste of time) is unprofitable and therefore it is assumed that in typical cases the number of observations in the range of 20 –30 is sufficient. Of course it is not necessary to repeat so many measurements manually – usually we can include such repetition into computer program.

When the number of observations is not large it is recommended to use the *Student's distribution* (t-distribution) instead of the normal distribution. The Students distribution is described by equation

$$p(t,\nu)=\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{t^2}{\nu}\right)^{-\frac{t+1}{2}} \qquad (2.68)$$

where $\nu = n\text{-}1$ is the degrees of freedom and Γ is the Euler function.

The shape of the graph of the Student's distribution is similar to the normal distribution (bell shape), but it is more flat and the flatness depends on the degrees of freedom (number of operations). Practically for the $n > 30$ the student's distribution is very close to the normal distribution.

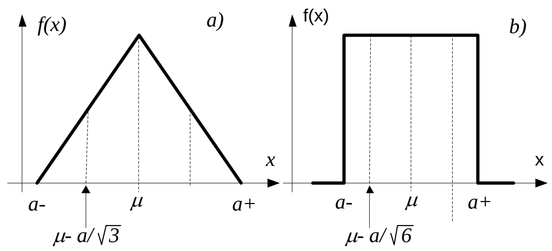


FIGURE 2.48
The graph of the triangular (a) and rectangular (b) density of probability function. There are indicated the range of variable corresponding with standard deviation 3σ of normal distribution

There are also other probability density functions, for example *rectangular (uniform) distribution* or *triangular distribution* presented in Fig. 2.17. We can assume that in the normal distribution the standard uncertainty is 1/3 of the range of variable equal to 3σ[13]. Note that in the case of triangular function the σ dispersion of normal distribution corresponds to the $1/\sqrt{3}$ of the range while in the case of rectangular function it is $1/\sqrt{6}$.

In the case when the resultant value is composed from various values $Y = c_1X_1+c_2X_2+...$ determined with various probability distributions the *Central Limit Theorem* is helpful. This theorem states that the distribution of Y will be approximately normal with expected value equal to:

$$\mu(Y)=\sum_{i=1}^{N} c_i\,\mu(X_i) \qquad (2.69)$$

and variance is:

$$\sigma^2(Y)=\sum_{i=1}^{N} c_i^2\sigma^2(X_i) \qquad (2.70)$$

In the case of evaluation of *uncertainty of type A* the calculations are relatively simple because we use well known tools of statistical analysis. The case of evaluation of *uncertainty of type B* is more complex, because we should evaluate various sources of uncertainty – for this task experience, knowledge and even intuition is necessary.

Relatively easy is evaluation of the uncertainty of typical measuring devices, because we have the information about the accuracy estimated by the manufacturer. Usually the reputable manufacturers enclose detailed documentation specifying all uncertainties. In the case of precise and expensive devices manufacturer can enclose the certificate of accuracy prepared by accredited laboratory.
The analogue indicating instruments are very well described by the standards, for example EN 60051 (EN 60051 1989). All instruments are divided into *Class of Accuracy CL*– for example 0.2, 0.5, 1, 2 etc. This class means that the absolute uncertainty Δx of all enumerating graduations does not exceed value $CL{\cdot}x_{max}$[14]. Thus

$$\textit{Accuracy Class} = \frac{|\Delta x|}{x_{max}} \qquad (2.71)$$

Thus the absolute uncertainty is the same for all measured values, while the relative uncertainty is smallest at the end of the range and equal to CL (Figure 2.49).

[13] We can assume that all measured value are in 3σ range with sufficient large probability.

[14] Sometimes the manufacturer encloses the table of corrections to all enumerated graduations.

$$\Delta x = x_{max} \cdot CL \tag{2.72}$$

while relative uncertainty is:

$$\delta x = \frac{\Delta x}{x} = \frac{x_{max}}{x} \cdot CL \tag{2.73}$$

Thus in the middle of scale the uncertainty is 2·CL and we should avoid measure below this point[15].

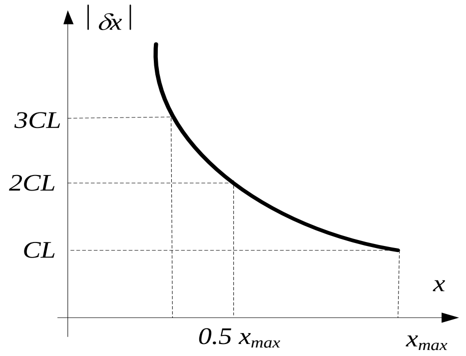


FIGURE 2.49
Dependence of relative uncertainty on the point on the scale of analogue instrument.

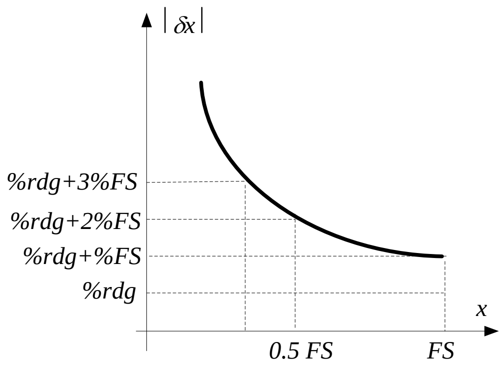


FIGURE 2.50
Dependence of relative uncertainty on the reading value of digital instrument.

The digital instruments are not standardized, but there is certain universally accepted custom of describing of the accuracy of such instruments. Usually the uncertainty of digital instruments is described as:

$$\pm(\%\, rdg + \%FS) \tag{2.74}$$

[15] Similar recommendation is in a car engine – we can drive on the lower gear but it is not reasonable.

which we can explain as the sum of uncertainty of indicated value (*rdg – reading*) and uncertainty of the range (*FS – Full Scale*). This absolute uncertainty is described using the unit of the measured value. The relative uncertainty is:

$$\delta x = \frac{\%rdg + \%FS}{rdg} \tag{2.75}$$

Thus for FS we obtain $\delta x = \%rdg + \%FS$, but for half of range (*rdg=1/2 FS*) we obtain $\delta x = \%rdg + \%2FS$ (Figure 2.50). We see that although method of describing of uncertainty is slightly other the effect is very similar to those in analogue instruments .

Why it is used so strange method of describing of uncertainty (instead of simply for example: uncertainty 0.002%)? It is why because this way it is possible to avoid inappropriate use of measuring instrument – for example if we have for digit high accuracy instrument and we read only two digits).

Consider case when four-digit voltmeter with the range *10V* indicated *0.499 V* and its uncertainty is described as ± (*0.05 + 0.01)%* . The uncertainty of the result is ± (*0.05%·499 + 0.01%·10 000)mV = ± 1.25 mV* and the relative uncertainty is ± *0.25%*. This example demonstrates importance of the use of all significant digits. If for example we change the range to *1V* (if such range exists) and we obtain the result *499.9 mV* the absolute uncertainty is ± *(0.05%·499.9 + 0.01%·1000 mV) = ± 0.35 mV* which is related to the uncertainty *0.07%.* Thus we improved the uncertainty more than three times only by changing the range of instrument.

Modern measuring devices are so accurate that the percent unit is too large for express the uncertainty. For example, presentation of the uncertainty as *0.00001%* would be inconvenient; therefore, often the description in *ppm* (*ppm – parts per million –* 10^{-6}) is used. The formula (2.74) is then presented as for example ±(*ppm reading + ppm range*).

TABLE 2.2
Resolution of a digital measuring instrument as the dependence on the number of digits used.

number of digits	number of counts	resolution
3-digit instrument	1000	0.1%
4-digit instrument	10 000	0.01%
4½-digit instrument	20 000	0.005%
4¾-digit instrument	50 000	0.002%

In 4½-digit instrument the first digit can be 0 or 1 and the rest 0,1,....9 while in the 4¾-digit instrument the first digit can be 0,1,2,3,4 and the rest 0,1,...9.

If we do not possess the documentation of measuring device we can adopt its accuracy basing on number of

digits – assuming that the producer design this instrument correctly. Table 2.2 presents information about the resolution of various digital instruments.
The parameters declared by producer are usually determined for the *nominal conditions* (for example temperature *23° C*, relative humidity *40 – 60%*, frequency *50 Hz*, etc.) Sometimes the *operating conditions* are recommended (for example variation of the temperature $\Delta T = 10°C$, inclinations from the horizontal level *5°*, etc.).

Additional problems appear when we process time varying signals. One of the important parameters in this case is the *frequency bandwidth* of the measuring instrument. Most of measuring devices exhibit limited bandwidth – sometimes it is limited for low frequency (the *DC* and slowly varying components are not detected), but it is always limited for high frequency. Figure 2.51 presents the specification of the measurement uncertainty of the popular multimeter 34401 of Agilent for various ranges of frequency of measured signals.

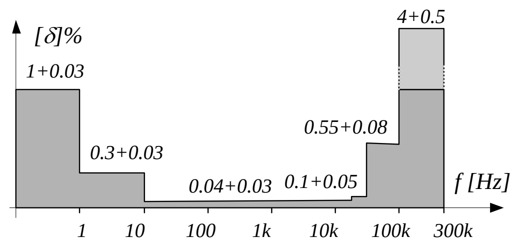


FIGURE 2.51
The example of specification of the measurement uncertainties depending on the frequency of the measured AC signal – the multimeter 34401 of Agilent.

The measurement of the signals at frequency other than acoustic range *20 Hz – 20 kHz* is in general less accurate. It is rather difficult to eliminate the influence of parasitic capacities at the high frequency range. Above about *1 MHz* the accuracy influences the transmission line effect and in this bandwidth special measuring instruments are used. In the case of analogue processors the bandwidth is usually defined as the frequency range in which the amplification factor K_u (or generally sensitivity coefficient *K*) changes no more than *3 dB* from the defined value (often for example from the value determined for frequency *1 kHz*).

In the case of digital processing the main limitations come from the *sampling frequency* which according to the Shannon theory should be at least two times larger than the greatest frequency in the signal. Recently, there are available analogue-to-digital converters with sampling frequency greater than *1 GHz*.

Sometimes the limitation can be characterized by acceptable the *CF* factor (*CF – Crest Factor*) – the ratio of the peak value to the *rms* value of the waveform. For example, in the multimeter 34401 presented above the Crest Factor of *1 – 2* causes additional error of *0.05%* of reading, while for *CF 4 –5* this error is *0.4%* of reading.

Sometimes the information "*True rms*" is placed on the front panel of measuring instruments. It means that the *rms* value of the signal is measured according to definition (*rms – Root Mean Square*):

$$\hat{x} = \sqrt{\frac{1}{T}\int_{t}^{t+T} x^2(t)dt} \tag{2.76}$$

The term "*True rms*" appeared as the reaction to the "non-true" measurements performed by the formerly universally used measuring instruments with rectifiers. Such instruments measure *de facto* the rectified average value, but they were scaled as *rms* devices under assumption that the dependence between these two values is the *Form Factor = 1.11*. But this condition is fulfilled only for pure sinusoidal waveforms, which is frequently not the case in typical measurements. For example, if the waveform is triangular the error resulting from the distortion is about *5.5%,* while for rectangular waveform this error is *11%*. And for pulse measurements this error is as large as around *50%* for the *crest factor = 4*. Recently the measuring devices indicated as "True *rms*" measure the distorted signal with CF up to *4* without any additional errors.

It is important to know that most laboratory multimeters do not measure the *rms* value of *AC+DC* signals. Usually the *AC* signals are separated from the input by a capacitor. Thus to obtain the *rms* value of *AC+DC* signal it is necessary to perform the measurement two times (as *DC* measurement and *AC* measurement) and then the resultant *rms* value can be calculated as

$$rms(AC + DC) = \sqrt{AC^2 + DC^2} \tag{2.77}$$

Currently many portable instruments are indicated as *AC+DC*. It means that these instruments correctly measure the *AC* signal with *DC* component.

We often use of measuring device transducers. In this case all specification related to converters, as nonlinearity, sensitivity, zero (see Chapter 2.2) drift should be considered as uncertainty type B. Often documentation informs about accuracy assuming that all errors are limited to declared value.

The error of dynamics can be described as the difference between the output signal *y(t)* and the steady value y_u (see Figure 2.18):

$$\Delta_d y = y(t) - y_u \tag{2.78}$$

or as the mean square value

$$\sigma_d = \sqrt{\int_0^{\infty} \left|\Delta_d y^2(t)\right| dt} \tag{2.79}$$

Sometimes it is convenient to analyze the error of dynamics not in the time domain but in the frequency domain. It is justified because in the case of linear circuit these both specifications are equivalent according to the *Parseval rule*

$$\int_0^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |f(j\omega)|^2 d\omega \tag{2.80}$$

Such considered dynamics errors (see Figure 2.24) can be then determined as:

$$\delta_d = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} |G(j\omega) - K|^2 |x(j\omega)|^2 d\omega} \tag{2.81}$$

After estimation of uncertainty type A and B we can start to prepare uncertainty budget. The Guide differentiates between the variance σ (an abstractive term) and *estimate of variance s* (*experimental standard deviation)* related to measurements. The experimental standard deviation is determined from the same dependence as the standard deviation (see Eq. 6.24):

$$s^2(x_k) = \frac{1}{n-1} \sum_{k=1}^{n} (x_k - \bar{x})^2 \tag{2.82}$$

The uncertainty of measurement is

$$u(\bar{x}) \equiv s(\bar{x}) \tag{2.83}$$

The result of measurement can be presented as

$$x = \bar{x} \pm u(\bar{x}) \tag{2.84}$$

The uncertainty of measurement can be also presented as the *expanded uncertainty u*

$$u = ku(\bar{x}) \tag{2.85}$$

where *k* is coverage factor (2 or 3).

Finally we calculate the resultant uncertainty determined using both methods

$$u(x) = \sqrt{u_A^2(\bar{x}) + u_B^2(x)} \tag{2.86}$$

Often the determined value is composed of many measurements. For example we test the electric power *P* by measure of current *I* with uncertainty *u(I)*, voltage *V* with uncertainty *u(V)* and phase shift $cos\varphi$ with uncertainty $u(cos\varphi)$. Next we calculate this power as:

$$P = VI\cos\varphi \tag{2.87}$$

Next we are interested to determine uncertainty of the final result – power *u(P)*. How to aggregate these component uncertainties?

When the determined value of *y* is function of several other quantities x_i

$$y = f(x_1, x_2 ... x_n) \tag{2.88}$$

we can determine the *combined uncertainty.* The estimation of combined uncertainty is more complicated when component quantities x_i, x_j are *mutually correlated* (dependent). We can check this by testing the *degree of correlation*

$$r(x_i, x_j) = \frac{u(x_i, x_j)}{u(x_i)u(x_j)} \tag{2.89}$$

which varies from *0* to *1* (*0* means that these quantities are uncorrelated, while *1* means that they are completely correlated, i.e. $x_i = k\, x_j$). In the dependence (2.89) $u(x_i)$ $u(x_j)$ are the estimates of variances, while $u(x_i,x_j)$ is the estimate of *covariance* of both quantities. We can determine the combined uncertainty using the *law of propagation of uncertainty*

$$u^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j) \tag{2.90}$$

The covariance of random variables can be determined experimentally as

$$u(\bar{x}_i, \bar{x}_j) = \frac{1}{n(n-1)} \sum_{k=1}^{n} \left(x_i^{(k)} - \bar{x}_i\right) \cdot \left(x_j^{(k)} - \bar{x}_j\right) \tag{2.91}$$

In practice, when the component quantities are mutually weakly dependent we can neglect the second part from the dependence (2.90) and the combined uncertainty can be calculated as

$$u^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \qquad (2.92)$$

If we would like to determine of the uncertainty of power measurement (Eq. 2.87) we should perform following calculation:

$$u(P) = \sqrt{(I\cos\varphi)^2 u^2(V) + (V\cos\varphi)^2 u^2(I) + (VI)^2 u^2(\cos\varphi)} \qquad (2.93)$$

Sometimes more convenient is to operate on relative values of uncertainty. In such case Eq. (2.92) should be converted to following relation:

$$\delta^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 \left(\frac{x_i}{y} \right)^2 \delta^2(x_i) \qquad (2.94)$$

Then after calculation the combined uncertainty is described by much simpler relation:

$$\delta(P) = \sqrt{\delta^2(V) + \delta^2(I) + \delta^2(\cos\varphi)} \qquad (2.95)$$ [16]

Sometimes instead of the root of square sum we simply add the modulus of uncertainty

$$u(y) = \sum_{i=1}^{N} \left| \frac{\partial f}{\partial x_i} U(x_i) \right| \qquad (2.96)$$

but such method of calculation of uncertainty can cause the overestimation of the combined uncertainty. The sum of modulus is sometimes called the *maximal limiting uncertainty*.

When the result of a measurement is presented, then the form of its presentation should inform us about the uncertainty as well. For example, if we have measured the voltage as *4.565 V* with uncertainty *0.1%* (thus it is *4.565±0.005*) then it is meaningless to present the result as for example *4.565297 V* (such style of presentation is sometimes met, when the researcher used the calculator or computer to estimate the results). Of course, similarly incorrect is to present the result as *4.56 V*. Generally, the accepted rule is that: *last significant digit of result of measurement should be the same range as the last digit of the uncertainty*.

2.6 Standards of electrical values - calibration

The measurement is always related to the standard unit. The *standard* is the realization of a given quantity with stated value and measurement uncertainty, used as a reference. Using the standard we can perform calibration of the measuring instrument. The *calibration* is the operation establishing the relationship between quantity values provided by measurement standards and the corresponding indications of measuring system, carried out under specified conditions and including evaluation of measurement uncertainty (ISO VIM 2004).

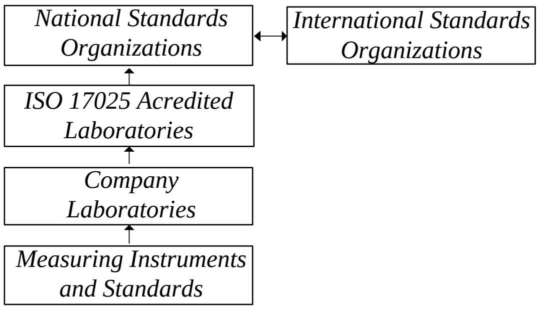


FIGURE 2.52
The example of traceability ladder

It is not necessary to compare our measured value directly with a standard because recently such comparison is often substitution by use of calibrated device. It means that producer of such measuring instrument or accredited laboratory guarantee that our instrument exhibits sufficient accuracy.

Figure 2.52 presents the ladder of standard organizations. On the top there are National Standard Organizations theoretically equipped with the best testing instruments and standards. These Organizations have supervision on Accredited Laboratories authorized to calibrate measuring instruments. These laboratories are governed by international standard "ISI/IEC 17025 – General requirements for the competence of testing and calibration laboratories". Producers of measuring instruments and companies where quality is tested by measuring instruments are obliged to periodically test and calibrate these instruments.

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[16] Now we can return to explanation of the equation (2.14). From the relation (2.12) $R_x = R_2 \frac{R_3}{R_4}$ we can easy calculate uncertainty of determination of resistance R_x. By using Eq. (2.94) we obtain the relation (2.14) $\delta R_x = \sqrt{(\delta R_2)^2 + (\delta R_3)^2 + (\delta R_4)^2}$.

There are several national or international organizations equipped with the best standards collaborating with National Standard Organizations: BIMP - International Bureau of Weights and Measures (French: Bureau international des poids et mesures), NIST - National Institute of Standards and Technology, PTB - Physikalisch-Technische Bundesanstalt or NPL - National Physical Laboratory (UK).

It is important to ensure unbroken chain between hierarchical standards – *traceability*. In this way it is possible to calibrate simple measuring instruments by laboratory instruments, these can be calibrated by high accuracy *reference instruments* or *calibrators*. At the top there are the best accuracy standards used to calibrate reference instruments.

When we say that the measurement requires a comparison to the standard value of measured quantity we do not need to apply the standard of this quantity. It would be impossible and impractical taking into account the great number of various quantities. Therefore it is sufficient to define and reproduce the standard of certain number of quantities (called *base quantities*) and next to derive other quantities as the *derivative quantities*. The derivative quantities can be determined from the mathematical dependencies deduced according to the physical laws and rules (for example to know the resistance of 1Ω it is only necessary to know the voltage $1\ V$ and current $1\ A$ according to the Ohm's law $1\Omega = 1V/1A$ – although just in this case standards of all quantities: voltage, resistance and current are available). Basing on this concept it was proposed to select seven base units in form of the *International System of Units - SI* (Table 2.3)[17]. From these seven basic units of SI we can derive other units – for example the discussed above unit of resistance *ohm*, Ω, can be expressed as: $m^2{\cdot}kg{\cdot}s^{-3}{\cdot}A^{-2}$.

The International System of Units was adopted by the General Conference on Weights and Measures CGPM and was described in ISO Standards: ISO 1000 – *SI units and recommendations for the use of their multiple as of certain other units* and ISO 31 – *Quantities and units*[18].

TABLE 2.3
Base quantities and base units of SI system.

base quantity	base unit	
name	name	symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Modern standards of highest accuracy are based on the quantum physics. The main advantage of such standards is that they are related to fundamental physical constant and can be reconstructed without reference to better sources. They even can be transferred to other civilization because they are nondestructive, as based on universal physical principles.

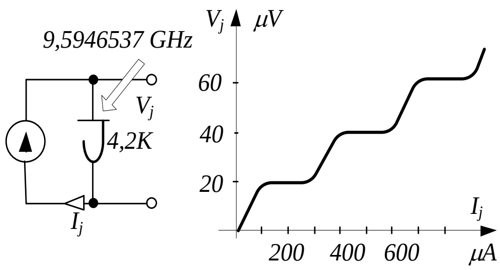

FIGURE 2.53
The Josephson quantum effect as the standard of voltage, after (Ibuca *et al* 1983)

The *quantum Josephson effect* is used to reconstruct the standard unit of voltage [Hamilton et al 1997, Kohlmann et al 2003, Benz et al 2004]. This effect appears at very low temperature (typically liquid helium *4.2 K*), when certain materials (for example niobium) become superconductors. The superconducting Josephson junction consists of two thin superconductors separated by very thin insulator layer.

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[17] The definitions of basic units are as follows:
One *meter* is equal to the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second;
One *kilogram* is equal to the mass of the international prototype of kilogram;
One *second* is a time interval equal in duration to 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom;
One *ampere* is equal to constant current which, if maintained in straight parallel conductors of infinite length and of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to $2{\cdot}10^{-7}$ N per each meter of length;
One *kelvin* is temperature equal to a fraction 1/273.16 of the thermodynamic temperature of the triple point of water;
One *mole* is the amount of the substance in a system, which contains as many elementary entities as there are atoms in 0.012 kg of carbon 12;
One *candela* is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540{\cdot}10^{12}$ Hz and that has a radiant intensity in that direction of 1/683 watt per steradian.

[18] The problems of electrical standards are the subject of interest of following institutions: BIMP (*International Bureau of Weights and Measures*), ISO (*International Organisation of Standardization*) and IEC (*International Electrotechnical Commission*).

When Josephson junction device is irradiated by microwave energy in the frequency range *70 – 100 GHz* and it is biased by *DC* current then the voltage changes stepwise with the change of the junction current (Figure 2.53). We can determine these steps on the volt-ampere curve very precisely. The level of the *n* step is described by the dependence:

$$U(n) = nf\,\frac{h}{2e} = \frac{nf}{K_J} \qquad (2.97)$$

Note that the voltage depends on the very well defined values: *h* – Planck's constant, *e* – electron charge and *f* – microwave frequency that we are able to measure very accurately. The Josephson's constant $K_J = (2e/h)$ is equal to *483 597.9 GHz/V*.

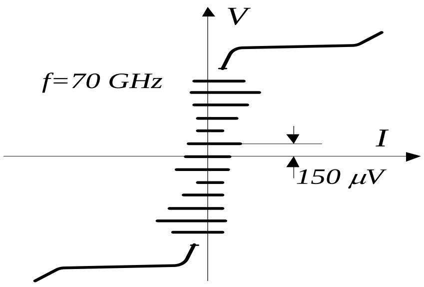

FIGURE 2.54
The Josephson quantum effect in underdamped junction.

The main drawback of the quantum standard of voltage is that the output signal is relatively small and contains noise. For *100 GHz* microwave the single step of voltage is about *200 μV*. This signal can be increased by connecting many Josephson junctions in series.

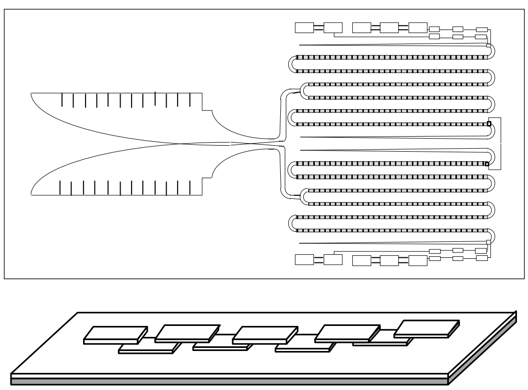

FIGURE 2.55
Typical design of array junction superconducting standard of 1 V: layout and microline.

Connecting of many junctions in array was complicated because every junction exhibited slightly other step current. This problem was solved by Lewinsen [Levinsen *et al* 1977] who proposed to underdamp the junction. The underdamped junction exhibits zero-crossing steps as it is presented in Figure 2.54. Thus it was possible to design junction array with bias current close to zero. Such arrays available also commercially consist typically of 2 400 or 13900 junctions for 1V or 10 V standards respectively (for electromagnetic field 75 GHz).

Figure 2.55 presents typical design of array junction standard of 1V. It is design of NIST composed of 3020 junctions. The meander shape strips create transmission microwave line. The junctions are created by deposition of niobium layer separated by oxidized aluminum Al_2O_3 layer of thickness of about several nm.

Commercially available standard of 10 V of Hypres Inc. is composed of 20 208 Josephson junctions every with area 18 × 38 μm supported by 72 – 78 GHz electromagnetic wave. Operating temperature is 4.2 K and operating power approximately 10 mW. Declared uncertainty is 0.05 ppm. It is estimated that possible is to construct such standard with uncertainty 10^{-10} – 1nV at 10 V[19].

Recently it can be observed effort to extend the idea of Josephson junction also on programmable DC voltage standard and AC voltage standards [Chevtchenko et al 2005]. Two concepts are considered – binary weighted arrays of junctions (this idea can be also useful for AD converters) and pulse driven arrays and next digital synthesis of arbitrary waveform and frequency. Unfortunately underdamped junctions (Figure 2.54) are hard to adapt for this purpose because they are hardly adjustable. Therefore designers returned to overdamped junctions (Figure 2.53). Fortunately problem of dispersion of bias currents for every junction is now not too dangerous because meantime technology and repeatability of junctions have been significantly improved.

The quantum Hall effect in the superconductors can be used to realize the standard of the resistance. It was discovered by Klaus von Klitzing in 1980. The specially prepared GaAs heterostructure with two-dimensional electron gas 2DEG was used as Hall device (Figure 2.56). When this device is placed in very small temperature (*1 – 2K*) and it is biased by the *DC* current the output Hall voltage depends on the magnetic field in a stepwise way (Figure 2.57).

[19] It is worth of attention that although we have excellent standard of voltage in the SI system electrical units are still represented by unit of current.

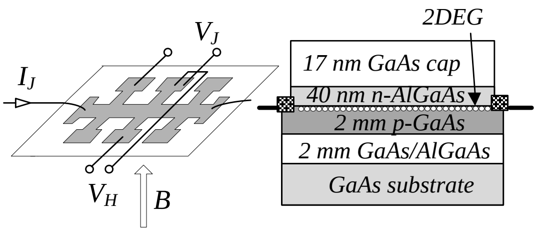

FIGURE 2.56
The quantum Hall effect device.

Typical *quantum Hall device* [Jeckelman *et al* 2001, Witt 1998] has six or eight terminals – two for Hall output voltage V_H , two for bias current I_J and two or four for voltage V_J . The voltage V_J is used to determine the current in the device. Moreover, the voltage V_J helps in determination of the step in the $R_H = f(B)$ characteristic, because maximum of V_J corresponds with each step (Figure 2.57). The Hall resistance for n^{th} step is $R_H(n) = V_H/I_J$.

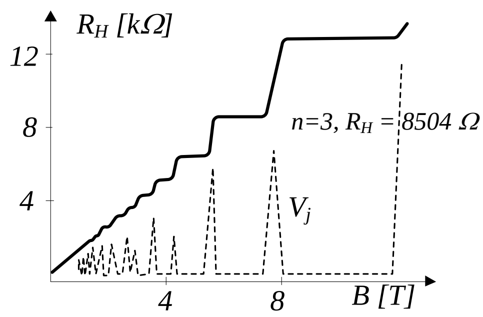

FIGURE 2.57
The quantum Hall effect as the standard of resistance.

The quantum Hall resistance of the n^{th} step is described by the dependence

$$R_H(n) = \frac{h}{2e^2 n} = \frac{K_K}{n} \qquad (2.98)$$

The resistance depends only on well-defined values (*h* and *e*) and it does not depend on the current I_J or the magnetic flux density *B*. The *von Klitzing constant* K_K was determined as *25 812.807 Ω*. Using the quantum resistance standard it is possible to reconstruct the resistance unit with uncertainty of about 10^{-3} *ppm*.

The important drawback of the quantum resistance standard is the very large magnetic field necessary to obtain the quantum phenomenon – for most often used forth step this field is several *T* (*T – tesla*).

A good candidate for quantum Hall device is grapheme because this material is inherently two-dimensional. Indeed the first experiments are promising – the quantum Hall effect was detectable in room temperature [Zhang et al 2005, Novoselov et al 2007].

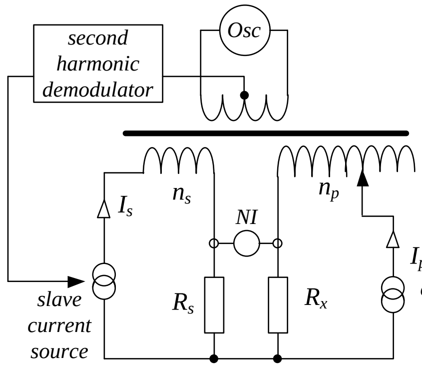

FIGURE 2.58
The direct current comparator used by NIST as the resistance standard (NIST1458 2003)

Figure 2.58 presents the commonly used method for the reconstruction of the resistance standard from the Hall quantum resistance device (NIST1458 2003).

In the current comparator presented in Figure 2.58 two simultaneous balances are required – ampere-turn balance and voltage balance. The ampere-turn balance is performed automatically using the feedback circuit to control the slave current source (as the state of balance the second harmonic induced in the transformer is used). Thus automatically is fulfilled the condition:

$$n_s I_s = n_p I_p \qquad (2.99)$$

The null-indicator detects the difference between the voltage drops on the resistances: standard R_s and measured R_x

$$R_x I_p = R_s I_s \qquad (2.100)$$

The condition of the balance is therefore:

$$R_x = \frac{n_p}{n_s} R_s \qquad (2.101)$$

The state of balance can be achieved by adjustment of the number of turns n_p. It enables the investigator to determine the measured resistance with excellent accuracy because the count of the number of turns is practically without error.

Frequency (and time) is the physical values measured with the best accuracy – it is reached uncertainty on the level of 10^{-16} [Bauch 2003]. According to the definition of the time unit one second is the duration of *9 192 631 770* cycles of microwave light absorbed or emitted by the hyperfine transition of cesium-133 atoms in their ground state undisturbed by external fields. This idea is realized as the time/frequency standard. The example of the cesium atomic standard is presented in Fig. 2.59.

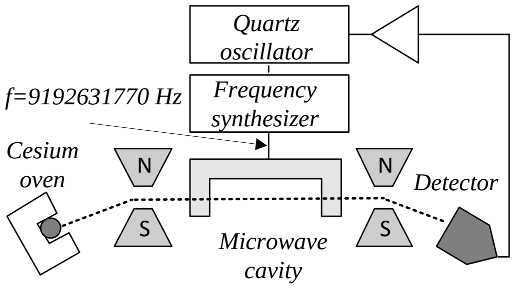

FIGURE 2.59
Cesium beam atomic clock – the standard of the time/frequency.

The Cesium-133 atoms are heated to the gaseous state in the oven. This gas is traveling as high-velocity beam through the gate of the magnet into the microwave cavity. The magnet gate is used to select only atoms of a particular energy state. The atoms are exposed in the cavity to a microwave frequency. If the microwave frequency matches the resonance frequency of cesium the atoms change their energy state. Only atoms which changed their energy pass through the second magnet gate. The detector of these atoms tunes the quartz oscillator to the state, at which the greatest number of atoms reaches the detector. It is when the frequency of microwave cavity is exactly *9 192 631 770 Hz*. This standard known as atomic clock exhibits uncertainty of about 10^{-13}.

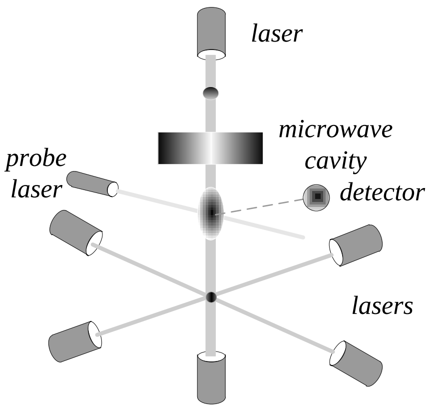

FIGURE 2.60
Fountain cesium atomic clock (NIST TF 2005)

Recently the most accurate atomic clock is achieved by means of fountain principle (Figure 2.60). Such clock developed by NIST allows to obtain an uncertainty better than 10^{-15} (NIST TF – 2005). This uncertainty is better than *0.1 ns/day*, which corresponds to a change around *1s* after *30 000 000* years.

Six infrared lasers orthogonally positioned (see Figure 2.60) in the vacuum chamber push the cesium atoms into a ball. In this process the lasers cool the atoms to the temperature a few millionths of a degree above absolute zero and reduce their thermal velocity to a few centimeters per second.

Vertical laser tosses the ball upward and then all the lasers are turned off. Under the influence of gravity the ball falls back through the microwave cavity. During this trip the atoms interact with the microwave signal. When the ball leaves the cavity another laser beam is directed onto the ball. Those atoms, whose states were altered, emit fluorescence sensed by detector. This process is repeated many times for various microwave frequency and the frequency that causes the maximum of fluorescence is the cesium resonance.

The superconducting quantum device SC SQUID [Tumanski 2011] convert external magnetic field in form of sinusoid of the period Φ_0:

$$\Phi_0 = \frac{h}{2e} = 2.067833667 \times 10^{-15} Wb \qquad (2.102)$$

We see that also magnetic unit (magnetic flux) is directly related to fundamental physical constant. But this unit is too small and SQUID device too complex to use it as standard of magnetic field. The SQUID is commonly used for measurement of extremely small magnetic fields (for example in biomagnetism) and for calibration of magnetometers usually is used other physical phenomenon – magnetic resonance [Weyand 2001, Park *et al* 2005].

The resonance frequency f_0 depends on magnetic field B:

$$f_0 = \gamma B \qquad (2.103)$$

and the gyromagnetic ratio γ is known with high accuracy. For proton ^{1}H resonance it is:

$$\gamma = 42.5760812 \quad MHz / T \qquad (2.104)$$

For high magnetic field resonance a deuterons ^{2}H can be used with gyromagnetic ratio:

$$\gamma = 6.535692 \quad MHz / T \qquad (2.105)$$

For weak magnetic field the electron resonance can be used with gyromagnetic ratio for ^4He:

$$\gamma = 28.02468\ GHz / T \tag{2.106}$$

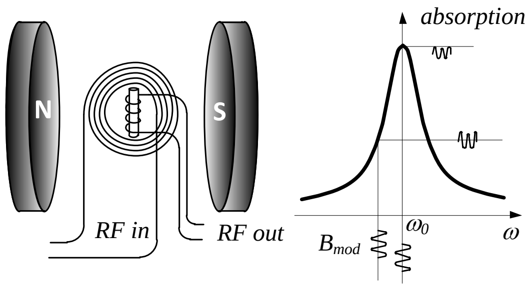

FIGURE 2.61
The nuclear magnetic resonance magnetometer.

Figure 2.61 presents typical NMR magnetometer principle. The resonance is detected by the sensing coil wound on the protons rich sample (for example with water). Frequency is tuned by the second coil perpendicular to sensing coil and to measured magnetic field. Sometimes additional modulation field is added to measured field – this way the state of resonance is detected as the largest second harmonic of signal of modulated frequency (Figure 2.61).

The market available magnetometer PT2026 of Metrolab enables measurement of magnetic field in range 0.2 – 20 T (in 10 subranges) with uncertainty 5 ppm and resolution 0.01 ppm.

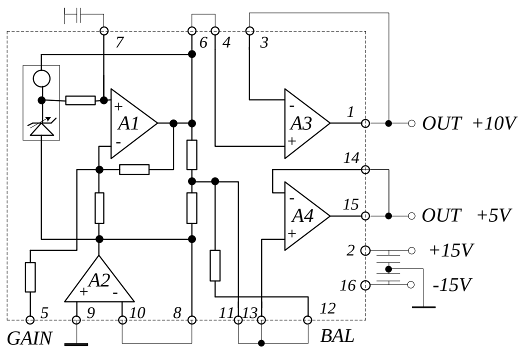

FIGURE 2.62
The functional block diagram of high precision voltage reference model AD588 of Analog Devices.

Beside described above the standards based on physical phenomena and physical constants in laboratories are used much simple and cheaper material references – simply prepared with high accuracy voltage source, resistors, capacitors etc.

Fig. 2.62 presents the functional block diagram of the voltage reference model AD588 of Analog Devices. This reference is designed with the accuracy suitable for 12-bit digital processing without any additional trimming elements. For better accuracy the trimming potentiometers can be used to adjust gain and balance.

The reference consists of precise laser trimmed Zener diode source and three additional amplifiers. It is possible to obtain the output voltage *+5 V, -5 V or 10 V* with uncertainty less than *1 mV*, temperature zero drift *1.5 ppm/K* and noises *6 μV p-p* in the bandwidth *0.1 – 10 Hz*. The stability of the output voltage is better than *15 ppm/1000 hours*. In the user notes (AD 2005) there are described among others the methods of application of this reference source to bridge circuit supply, in order to obtain precision current source or to excitation of the resistive temperature detector (sensor) *RTD*. Thus it is possible to obtain relatively cheap and useful standard reference voltage with the voltage uncertainty about 0.01%, suitable for many measuring purposes.

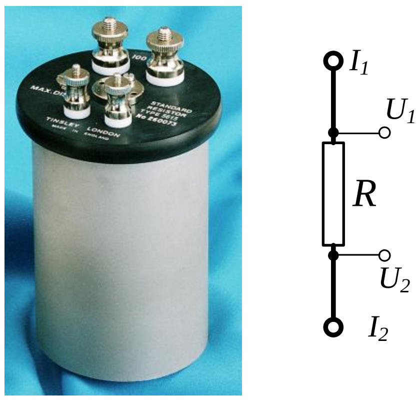

FIGURE 2.63
An example of the standard resistor – model 5615 of Tinsley (Tinsley 2005) (permission of Tinsley Precise Instruments).

The standards of resistance are manufactured as the precise resistors prepared as the wire or strip wound on the porcelain cylinder. Such resistor is placed usually in a shielded housing. The resistance standard is usually equipped with four terminals: two (larger) terminals are used for the current excitation and second two (smaller) ones are used as the voltage (potential) terminals (Figure 2.63).

There are certain materials suitable for precise resistors manufacturing – they should exhibit large resistivity and very small dependence of the temperature. One of the most popular is *manganin* (alloy of *84% Cu, 12% Mn, 4% Ni*) with resistivity $\rho = 0.42\ \mu\Omega \cdot m$ and temperature coefficient $\alpha_T = (0.5 - 2)\ 10^{-5}/K$ (for comparison pure copper exhibits the temperature coefficient $\alpha_T = 4{\cdot}10^{-3}/K$). Another material used for precise resistors preparation is the *Evanohm* (*75% Ni, 20% Cr, 2.5% Al, 2.5% Cu*) with resistivity $\rho = 1.2\ \mu\Omega{\cdot}m$ and excellent temperature properties (α_T less than $10^{-6}/K$). An important requirement for resistive materials is negligible thermoelectric voltage in relation to copper.

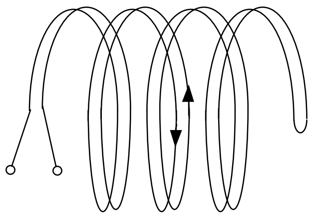

FIGURE 2.64
The principle of bifilar winding.

The resistors are wound bifilarly which enables users to obtain a resistor practically without the inductance. The bifilar winding (Figure 2.64) is performed in such a way that in adjacent wires the currents are in the opposite directions and therefore the magnetic fields compensate each other. The capacitance of standard resistor is very small due to special design. Although standard resistors are with negligible inductance and capacitance usually they are used for *DC* circuits. For *AC* applications special kind of resistors (indicated *DC/AC* resistor) can be used but in the bandwidth limited to about *1 kHz.*

Standards resistors available on the market exhibit typical uncertainty of *0.5 – 10 ppm*, long term stability of about *2 ppm/year* and temperature stability of about *2 ppm/°C*.

FIGURE 2.65
An example of the resistor decade box.

For everyday applications sometimes more usable can be adjustable *resistor decade box*. In such resistor is it possible to set desired value of resistance in the decade sequence: $x{\times}1000\Omega + x{\times}100\Omega + x{\times}10\Omega + \ldots\ldots$ Currently, there are available resistors with adjustable resistance in range $1\ m\Omega – 100\ M\Omega$. The smallest step is $1\ m\Omega$. The uncertainty of the resistor decade boxes is typically of *0.01%*. It should be noted that often such uncertainty is attainable for the total resistance of the resistor and with decreasing of the decade step it gradually increases (for example: *R/step 0.001 Ω - 4%, 0.01 Ω - 2%, 0.1 Ω - 0.4%, 1 Ω - 0.1%, 10 Ω - 0.04%,* the rest of resistors *0.01%* (IET Labs 2005). Fig. 2.65 presents the typical design of resistor decade box.

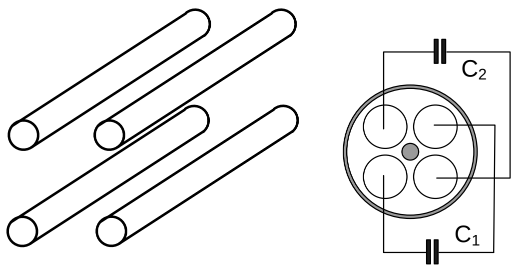

FIGURE 2.66
The standard of capacitance – calculable capacitor of Thompson-Lampard.

Thompson and Lampard proposed the standard of capacity based on electrostatic theory [Thomson *et al* 1956]. The capacitor proposed by *Thompson and Lampard* consists of four metal cylinder bars arranged in a square and surrounded by a cylinder movable shield device (Figure 2.66). The capacitance of such capacitors can be estimated from the following dependence

$$C = \frac{\ln 2}{4\pi^2}\frac{l}{c^2}10^7 \tag{2.107}$$

Thus, we can calculate the capacitance with very small uncertainty because it depends only on the velocity of light *c* (which we know very precisely) and on the length *l*. And the length we are also able to measure with very small uncertainty – using the interference methods. That is why we are able to determine such capacitance with uncertainty less than 10^{-2} *ppm*. The main drawback of the air standard capacitance is its relatively small capacitance – only *1.95354904 pF/m*.

It is possible to transfer the value of calculated capacitor to standards of other values – for example standard resistance (by comparison of both standards using special bridge circuit) [Jeffery 1997]. The Thompson-Lampard capacitor is an example of so called *calculable standards*. It means that based on theory we are able to determine capacity, inductivity or other values of well-defined geometry.

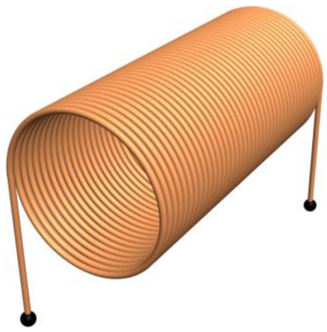

FIGURE 2.67
The air-cored coil as the calculable standard of inductance.

Theoretically also inductivity can be calculable standard. For example inductivity of the air core coil with n turns, radius r and length l is:

$$L = \frac{4\pi^2 n^2 r^2}{l} 10^{-7} \tag{2.108}$$

But due to finite value of the resistance it is practically not possible to obtain the ideal inductance standard. Also, it is not possible to eliminate the self-capacitance of the coil. Therefore the best solution is to construct coil and next to determine its inductance by reference meter. The inductance is usually described for defined value of frequency, most often for *1 kHz*.

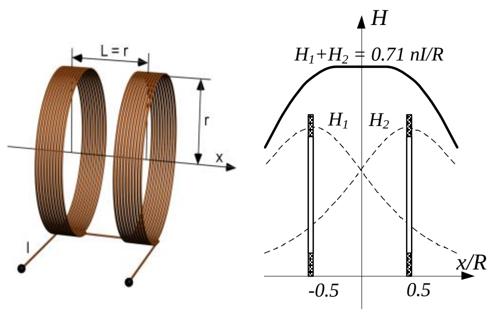

FIGURE 2.68
The Helmholtz coil as a standard of magnetic field.

As a standard of magnetic field a coil system known as a Helmholtz coil is used. For a special geometry design (distance between coils equal to their radius) we obtain the magnetic field uniform and the value of this field in the center depends only on dimension and current:

$$H = 0.7155 \frac{nI}{r} \tag{2.109}$$

It is relative easy to obtain *standard value of frequency*. As the standard of frequency we can use the *quartz oscillator*. Appropriately prepared quartz crystal can exhibit the stability of resonance frequency better than 10^{-8}*/year*. Unfortunately this frequency depends on the temperature; therefore it is necessary to use the thermostat. It is very convenient that the standard frequency can be transmitted by the radio. The frequency standard, model 910R of Fluke controlled by the satellite signal (cesium atomic standard in the GPS system) enables users to obtain the frequency with the stability better than 10^{-12}*/24 hours*.

There are also special digital measuring instruments with extremely small uncertainty. Such instruments can be used as the working standards. For example *reference multimeter* model 8508 of Fluke enables to measure the DC voltage with uncertainty *(0.7+0.5)ppm*, direct current with uncertainty *(6+2)ppm*, AC voltage with uncertainty *(50+10)ppm*, AC current with uncertainty *(200+100)ppm* and the resistance with uncertainty *(1+0.25)ppm*.

FIGURE 2.69
Calibrator model 5520 of Fluke (Fluke 2005) (permission of Fluke Corporation).

As the real working standards the measuring instruments called *calibrators* can be used. Such instruments can deliver the standard signals or values enabling to scale other measuring devices. As an example we can consider the *High Performance Multi-Product Calibrator* model 5520 of Fluke presented in Figure 2.69.

The performances of Fluke calibrator are presented in Table 2.4. This calibrator consists of two independent standard sources of *DC* and *AC* voltages or currents with controlled frequency and phase between them (for calibration of power meters and energy meters). Additionally calibrator can work as the standard resistance, inductance, capacitance and temperature (for modelling of thermoresistors or thermocouples). Thus this calibrator can deliver the reference values formerly available only by the high quality standards.

TABLE 2.4
The performances of calibrator 5520 model of Fluke.

Functions	Ranges	Uncertainty (95% 1 year)
DC voltage	0 - ±1020 V	12 ppm
AC voltage, 10 Hz - 500 kHz	1mV - 1020 V	120 ppm
DC current	0 - ±20,5 A	100 ppm
AC current, 10 Hz - 30 kHz	29 mA - 20,5 A	600 ppm
Resistance	0 - 1100 MΩ	28 ppm
Capacitance	0,19 nF - 110 mF	0.25%
Phase between AC signals	0 - ± 179,99°	±0.07°
Frequency	0,01 Hz - 2 MHz	25 ppm
DC power	10,9μW-20,5 kW	0.023%
AC power	10,9μW- 20,5 kW	0.08%
Thermocouple	-250°C - 2316°C	0.14°C
Thermoresistor	-200°C - 630°C	0.03°C

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